

# ON INTEGRALS OF THE CERTAIN ORDINARY DIFFERENTIAL EQUATIONS IN THE VICINITY OF THE SINGULARITY. II.

By

Minoru URABE

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## § 1. Introduction.

In our previous paper <sup>(1)</sup>, under generalized Poincaré's condition or Picard's condition, we have solved the equation as follows:

$$(1.1) \quad \sum_{i=1}^n X_i \frac{\partial f}{\partial x_i} = 0,$$

where  $X_i(x)$  are regular in the vicinity of  $x_i=0$  and are expanded as follows:

$$X_i(x) = \sum_{j=1}^n a_{ij} x_j + \text{sum of the terms of the second and higher orders.}$$

Let the eigen values of the matrix  $A = \| a_{ij} \|$  be  $\lambda_i$ . The generalized Poincaré's condition is as follows: *all  $\lambda_i$ 's lie in a convex domain  $\Omega$  which does not contain the origin.* The generalized Picard's condition is as follows: *some of  $\lambda_i$  lie in a convex domain  $\Omega$  which does not contain the origin.* In this paper, making use of solutions of (1.1), we are going to obtain integrals of the ordinary differential equations as follows;

$$(1.2) \quad \frac{dx_1}{X_1} = \frac{dx_2}{X_2} = \dots = \frac{dx_n}{X_n}.$$

By our previous paper <sup>(2)</sup>, without loss of generality, we can assume that  $A$  is of Jordan's form and that the real parts of  $\lambda_i$  lying in  $\Omega$  are positive. As in (A), we write  $X_i$  as follows:

$$X_{ip}^i = \lambda_i x_{ip}^i + \delta_p x_{ip-1}^i + \dots,$$

where the unwritten terms are of the second and higher orders, and  $\delta_p$  is equal to 1 or zero according as  $p \geq 2$  or  $p=1$ .

1) M. Urabe, *On Solutions of the Linear Homogeneous Partial Differential Equation in the Vicinity of the Singularity, III.* This Journal, Vol. 15, No. 1, p. 25. In the following, we denote this paper by (A).

2) M. Urabe, *On Integrals of the Certain Ordinary Differential Equations in the Vicinity of the Singularity, I.* This Journal, Vol. 14, No. 3, p. 209. In the following, we denote this paper by (B).