Reduced Forms of Ordinary Differential Equations in the Vicinity of the Singularity of the Second Kind.

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§ 1. Introduction.

The differential equation $\frac{dy}{dx}=f(y, x)$, in the vicinity of the singularity of the second kind $(y=0,\ x=0)$, by means of Weierstrass' preparation theorem, can be reduced to the equation of the following form[1]

$$(1.1)\qquad \frac{dy}{dx}=x^S y^R \frac{q(y,x)}{p(y,x)} e^{G(y,x)}$$

where S and R are integers, $G(y, x)$ is regular in the vicinity of $(y=0,\ x=0)$, $p(y, x)$ and $q(y, x)$ are relatively prime singular algebroid polynomial with the vertex $(y=0,\ x=0)$, and $p(0, x)$, $q(0, x)$ do not vanish identically. It happens that the function $p(y, x)$ or $q(y, x)$ becomes a constant, but, even in that case, $(y=0,\ x=0)$ is the singularity of the second kind of the differential equation.

Forsyth has shown that, if the equation (1.1) has a determinate integral of regular class,[2] then, by means of Newton's polygon method, it can be reduced to the equation of the type either

$$\text{I}:\quad t^{\kappa}\frac{dv}{dt}=av^n+pt+\ldots., \qquad \text{or} \qquad \text{II}:\quad t^{\kappa}\frac{dv}{dt}=\frac{av^n+pt+\ldots\ldots}{bv^m+qt+\ldots\ldots},$$

where κ, m, n are positive-integers and $a, b\neq 0$.[3] Besides, he has proved that, the equation of the type II which has an integral of regular class, is reduced to the equation of the type I, when certain conditions upon the coefficients are satisfied.[4]

In this paper, by means of Newton's polygon method, we shall establish the general theory of reduction of the differential equation in the vicinity of the singularity of the second kind. Without any assumptions which Forsyth has laid, we shall obtain the final reduced forms as follows:

$$(\mathrm{A}_1)\qquad t^{\kappa}\frac{dv}{dt}=av+pt+\ldots.; \qquad \kappa\geqq 2,\ a\neq 0,$$

$$(\mathrm{A}_2)\qquad t\frac{dv}{dt}=v^{\lambda}(av^n+pt^m+\ldots.); \qquad a,p\neq 0,\ n\geqq 0,\ m>0, \text{ and } \lambda\geqq 1 \text{ when } n=0,$$

$$(\mathrm{B}_0)\qquad t\frac{dv}{dt}=\frac{av^n+pt+\ldots\ldots}{bv^m+qt+\ldots.}; \qquad a,b\neq 0,\ m\geqq 1,\ n\geqq 1,$$

$$(\mathrm{C}_0)\qquad \frac{dv}{dt}=t^{\nu}v^{\lambda}(av^n+pt^m+\ldots.); \qquad a,p\neq 0,\ n\geqq 0,\ m>0,\ \nu\geqq 0,\ \lambda\geqq 1.$$

1) A.R. Forsyth, Theory of Differential Equations, Part II, p. 88.

2) When an integral has an appropriate order in powers of the independent variable in the vicinity of the initial values, we call such an integral "an integral of regular class."

3) Forsyth, ibid. p. 123.

4) Forsyth, ibid. p. 204.