SOME GENERAL THEOREMS AND CONVERGENCE THEOREMS IN VECTOR LATTICES

By

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Several attempts have been made to investigate the fundamental properties of vector lattice. Among them most important are the spectral theory developed by F. Riesz(1) and H. Freudental(2), and the representation theory by S. kakutani(3), M.H. Stone(4) and others. In my previous works I investigated the general properties of vector lattice, representation theories of vector lattice and linear operators. The purpose of the present paper is to make some additional remarks on my previous works(5) (Part 1) and to study the convergence character of linear operators with range in vector lattice (Part 2). Especially in § 5 (Part 1) I shall introduce a complex Banach lattice $Z=\{X, X\}$ and show that the properties of Z are reduced to those of its component real Banach lattice X, e.g., Z is reflexive if and only if X is reflexive.

Part 1 Some General Theorems on Vector Lattice

§ 1. Remarks on (o)-bounded linear operators. Let X be a vector lattice and Y a complete vector lattice. Let T be an (o)-bounded linear operator from X to Y. If $x_n \to 0$ (o) implies $T.x_n \to 0$ (o), then we say that T is (o)-continuous. If in this statement directed sets play a roll instead of simple sequences, that is, $x_\delta \to 0$ (o) implies $T.x_\delta \to 0$ (o), then we say that T is (o)- continuous in the sense of Moore and Smith, or MS-continuous.

THEOREM I. 1. If T is (o)-continuous, then so are $T_+, T_-, |T|$.

PROOF. Let $x_n \downarrow 0$, and put $y = \bigwedge_n T_+.x_n$, then $T_+.x_n - T.(a \frown x_n) \leqq T_+.x_1 - T.a$ for $0 \leqq a \leqq x_1$. By making use of this inequality we obtain $y \leqq T_+.x_1 - T.a$, since $a \frown x_n \downarrow 0$ and T is (o)-continuous. But by definition $T_+.x_1 = \bigvee_{0 \leqq a \leqq x_1} T.a$, and so $y = 0$.

THEOREM I. 2. If T is MS-continuous, then so are T_+, T_-, $|T|$.

The proof is very similar to that of Theorm I. I.

By an ideal J of X we mean a linear subset of X such that $|u| \leqq |v|$, $v \varepsilon J$ implies $u \varepsilon J$, and by a normal ideal the totality of elements of X orthogonal to each element of some subset of X. A normal ideal N of a complete vector lattice is characterized as a direct component of this vector lattice, or as such an ideal that l. u. b. of a subset of N,

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