THEORY OF THE SPHERICALLY SYMMETRIC SPACE-TIMES. V.
n DIMENSIONAL SPHERICALLY SYMMETRIC SPACE-TIMES

By

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In this paper we shall give an $n(\geqq 5)$ dimensional generalization of the theory of the spherically symmetric space-times developed by the present writer.[1] Almost all properties of the four dimensional S_0 given in the previous papers also hold for the n dimensional case to within some slight modifications, but as will be seen in due course, the theory concerning the latter is simpler than the four dimensional theory in some respects owing to the condition n-$2\geqq 3$. In this paper, emphasizing the properties which do not hold for the four dimensional S_0, we shall give the main results.

§1. Definition of S_0 and identities concerning c.s.

Definition: An $n(\geqq 5)$ dimensional s.s. space-time S_0 is an n dimensional Riemannian space having the following properties:

(I) Its curvature tensor satisfies

$$K_{ijlm}=\overset{1}{\rho}\alpha_{[i}\alpha_{[l}\beta_{j]}\beta_{m]}+\overset{2}{\rho}g_{[i[l}\alpha_{j]}\alpha_{m]}+\overset{3}{\rho}g_{[i[l}\beta_{j]}\beta_{m]}+\overset{4}{\rho}g_{[i[l}g_{j]m]}\,, \tag{F$_1$}$$

where $i, j, \cdots=1, \cdots, n$, α_i and β_i are mutually orthogonal unit vectors satisfying

$$\nabla_i\alpha_j=\sigma\alpha_i\beta_j+\kappa(g_{ij}-\alpha_i\alpha_j-\beta_i\beta_j)-\bar{\sigma}\beta_i\beta_j\,, \tag{F$_2$}$$

$$\nabla_i\beta_j=\bar{\sigma}\beta_i\alpha_j+\bar{\kappa}(g_{ij}-\alpha_i\alpha_j-\beta_i\beta_j)-\sigma\alpha_i\alpha_j\,, \tag{F$_3$}$$

and $\overset{a}{\rho}$; $\sigma, \bar{\sigma}$; $\kappa, \bar{\kappa}$ are scalars determined from these equations.

(II) One of the five scalars $\overset{1}{\rho}, \cdots, \overset{4}{\rho}$ and K is such that its gradient vector is a linear combination of α_i and β_i.

(III)
$$\overset{4}{\rho}+2(\kappa^2+\bar{\kappa}^2)\neq 0\,. \tag{F$_4$}$$

(IV) Moreover for the sake of simplicity and symmetry, we shall assume that the fundamental form is positive definite and $\alpha_s\alpha^s=\beta_s\beta^s=1$.

Corresponding theory for the case of g_{ij} of the type $(--\cdots+)$ and $-\alpha_s\alpha^s=\beta_s\beta^s=1$ is obtained by making some slight modifications. Through-