ITERATION OF CERTAIN FINITE TRANSFORMATION.

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Preface.

Given the finite transformation of the form as follows:

$$(0.1) \qquad T: \quad 'x^{\nu}=\varphi^{\nu}(x)=a^{\nu}_{\mu}x^{\mu}+a^{\nu}_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2}+\cdots\cdots .^{(1)}$$

We consider the case where the absoulte values of all the eigen values λ_ν of $\| a^{\nu}_{\mu} \|$ are unity. In the previous paper[2], it was shown that, when T is majorized, the equations of Schröder for T can be solved and, that, by means of their solutions, T is reduced to the linear transformation such that $'x^{\nu}=\lambda_\nu x^{\nu}$ (not summed by ν). When the arguments of the eigen values are all commensurable with 2π, by iteration of certain times, T can be reduced to the transformation where the eigen values are all unity. In this paper, we deal with this case. When T is majorized, the reduced transformation becomes an identical transformation[3]. In this paper, we do not assume the condition of majorizedness. By effecting a suitable linear transformation of the variables x^{ν}, without loss of generality, we may assume that $\| a^{\nu}_{\mu} \|$ is of Jordan's form. Thus the transformation which becomes a subject in this paper, may be written as follows:

$$(0.2) \qquad T: \quad 'x^{\nu}=\varphi^{\nu}(x)=x^{\nu}+\delta_{\nu-1}x^{\nu-1}+a^{\nu}_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2}+\cdots\cdots ,$$

where $\delta_{\nu-1}=0$ or 1.

In the case of one variable, the characters of the transformation of the form (0.2) were studied by means of iteration[4], and, in the neighborhood of the origin, there was found a domain, of which all the points converge to the origin always remaining in it when the transformation is

1) $a^{\nu}_{\mu}x^{\mu}$, $a^{\nu}_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2}$, ... mean that $\sum_{\mu} a^{\nu}_{\mu}x^{\mu}$, $\sum_{\mu_1,\mu_2} a^{\nu}_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2}$, ... In the following, we use this convention of tensor calculus.

2) M. Urabe, ***Application of majorized group of transformations to functional equations***, this Journal Vol. 16, No. 2 (1952), pp. 267-283.

3) do.

4) M. P. Fatou, Bull. Soc. Math. (1919).
G. Julia, Jour. Math. Pures Appl. (1918).
J. Malmquist, 1er mémoire, Ark. Mat. Ast. Fys. (1921).