ON THE DERIVATIONS AND THE RELATIVE DIFFERENTS IN COMMUTATIVE FIELDS

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In the previous publication [1],[1] I have, in a direct way, proved the equivalence of the definition by A. Weil and Y. Kawada of the relative different in algebraic number field with the usual one which was introduced by R. Dedekind.

Recently Prof. M. Moriya[2] has developed the theory of derivations in Noetherian ring and has given the definition of the relative different in such a commutative fields as, in its integral domains, the fundamental theorem of the multiplicative ideal theory holds. And, using the theory of $\mathfrak{p}$-adic fields, he has shown the equivalence of both definitions of the relative different in such a field.

Now, in the first step of this paper, we shall show the equivalence of both definitions without using the theory of $\mathfrak{p}$-adic fields (in the analogous way to my previous paper). In the next step, we shall show that the chain-theorem and the different-theorem hold independently of the usual one in the relative different defined by derivations.

1. Let $\mathfrak{R}$ be a commutative ring, then we consider for an ideal $\mathfrak{A}(\neq(0))$ in $\mathfrak{R}$ the residue class ring $\mathfrak{R}/\mathfrak{A}$ and define a *derivation* modulo $\mathfrak{A}$ in a subring $\mathfrak{R}'$ of $\mathfrak{R}$ as a unique mapping of $\mathfrak{R}'$ into $\mathfrak{R}/\mathfrak{A}$ with the following properties:

(1) D is a module homomorphism of $\mathfrak{R}'$ into $\mathfrak{R}/\mathfrak{A}$ i.e. for $\alpha, \beta \in \mathfrak{R}'$

$$D(\alpha+\beta)=D(\alpha)+D(\beta),$$

(2) for $\alpha, \beta \in \mathfrak{R}'$

$$D(\alpha\beta)=\beta D(\alpha)+\alpha D(\beta).$$

Now we denote by $\mathfrak{D}(\mathfrak{R}', \mathfrak{r}; \mathfrak{R}/\mathfrak{A})$ the totality of all derivations modulo $\mathfrak{A}$ in $\mathfrak{R}'$ which map every element in a subring $\mathfrak{r}$ of $\mathfrak{R}'$ onto the null element

1) The number in square brackets refer to the list of references at the end of this paper. See A. Kinohara [1].

2) See M. Moriya [1].