SEMI-MODULARITY IN RELATIVELY ATOMIC, UPPER CONTINUOUS LATTICES

By

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K. Menger [1][1] has introduced the following conditions to characterize the lattice of all subspaces of a finite dimensional affine space:

(η'') *If* p *is a point, then either* $p\leqq a$ *or* $a\cup p$ *covers* a, *for any element* a.

($\bar{\eta}$) *If* h *is covered by* 1, *then either* $a\leqq h$ *or* a *covers* $a\cap h$, *for any element* a *with* $a\cap h\neq 0$.

L. R. Wilcox [1] has shown that the lattice of all subspaces of an affine space is semi-modular in the sense that

(A) $(b, c)M$, $b\cap c=0$ *imply* $(c, b)M$, and

(B) $b\cap c\neq 0$ *implies* $(b, c)M$.

The semi-modularity in this sense was used in my previous paper [1] to characterize the lattice of all subspaces of an affine space of arbitrary dimensions, noting that it might be replaced by the following conditions:

(ξ') *If* a, b *cover* c *and* $a\neq b$, *then* $a\cup b$ *covers* a *and* b.

(P) *If* $p\leqq q\cup a$, $r\leqq a$, *where* p, q, r *are points and* a *is any element, then there exists a point* s *with* $p\leqq q\cup r\cup s$, $s\leqq a$.

While L. R. Wilcox [2] has shown that in a lattice of finite dimensions, (ξ') is equivalent to the condition:

(α) $(b, c)M$ *implies* $(c, b)M$,

which follows immediately from (A) and (B).

The purpose of this paper is to show that in any relatively atomic, upper continuous lattice, (ξ') is equivalent to (α), and also the combined conditions "(η'') *and* ($\bar{\eta}$)", "(A) *and* (B)", and "(ξ') *and* (P)" are equivalent to each other.

1. We begin by listing the definitions and several known lemmas we shall employ.

DEFINITION 1. A lattice with 0 is called *relatively atomic* if $a<b$ implies $a<a\cup p\leqq b$ for some point p.

1) The numbers in square brackets refer to the list of references at the end of the paper.