A LATTICE FORMULATION FOR ALGEBRAIC AND TRANSCENDENTAL EXTENTIONS IN ABSTRACT ALGEBRAS

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MacLane [1][1] has proved that the relatively algebraically closed subfields of any field Ω form a matroid lattice[2], and the transcendence degree is obtained as a lattice-theoretic dimension in this matroid lattice. But he did not consider the lattice-theoretic character of algebraic and transcendental extensions of subfields of Ω. On the other hand, in the lattice of convex sets there are two cases of the adjunctions of a point p to a convex set A, namely (1) p is contained in the least linear set $\bar{A}$ containing A, and (2) p is not contained in $\bar{A}$. In the case (1) the convex set obtained by the adjunction of p to A has the same dimension as A, but in the case (2) by the adjunction the dimension increases. These two cases are similar to a simple algebraic extension and a simple transcendental extension of subfields respectively. Hence we may expect that there exists a lattice theory which contains both the theory of extensions of fields and the theory of extensions of convex sets.

In this paper, I first find a Φ-relatively molecular, upper continuous lattice L such that the lattice of all subalgebras of an abstract algebra with finitary operations and lattice of all subgeometries of an abstract geometry with finitary operations[3] are special cases of such a lattice L, where Φ is the set of principal subalgebras and the set of principal subgeometries, that is points, respectively. Introducing in Φ the dependence relation formulated by van der Waerden [1, p. 204], I investigate the theory of extensions in L, with a view to showing that the theory of extension of fields and the theory of extension of convex sets are the special cases of this general theory.

1) The numbers is square brackets refer to the list of references at the end of this paper.

2) MacLane used "exchange lattice" instead of "matroid lattice", but these two conceptions are equivalent. Cf. Maeda [1] 181.

3) For the abstract geometry with finitary operations, cf. Maeda [1].