ON AXIOM OF BETWEENNESS

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Introduction

The system A, in which a binary relation (xy) (or ρ) satisfying $P(\equiv(P1, P2, P3))$ is defined, is called a partially ordered set and is written by $A[\rho, P]$. And the system A, in which a ternary relation (xyz) (or τ) satisfying $Q\equiv(Q_1Q_2......)$ is defined in A, is written by $A[\tau, Q]$. There is a problem concerning that $A[\rho, P]$ may be characterized by $A[\tau, Q]$, that is, the condition for Q in order that a binary relation ρ may be characterized by a ternary relation τ.

In this paper we shall investigate the case where the ternary relation τ becomes a betweenness. So our problem is to consider about the condition for Q, in order that a binary relation ρ (or order) satisfying P in A may be introduced from a subset $R[A, Q]$ satisfying Q in triple product space $A\times A\times A$, such that

$$\{xyz\}\in R[A, Q] \rightleftarrows x\rho y\rho z \quad \text{or} \quad z\rho y\rho x .$$

In other words, we purpose to investigate a complete system of axioms which characterizes "*betweenness*" in a partially ordered set A. G. Birkhoff[1], E. Pitcher and M. F. Smiley[2] have discussed about the necessary conditions for betweenness.

In chapter I, we consider about the closed betweenness[3] of a partially ordered set A. In §1, we have necessary conditions B1, B2, B3, B4, B5, B6 and B7, where B3$\equiv$(1), B4$\equiv$(2), B5$\equiv$(4) in the necessary conditions (1), (2), (3), (4) and (5) proposed by G. Birkhoff, and in §2 we inquire into the independency of them. And in §3, 4, 5, 6, 7 and 8, by considering the decomposition of the set A, by means of a collection of subsets of A: S and by the distance introduced from S, we discuss their sufficiency for closed betweenness of set A and investigate the uniqueness concerning their ordering.

1), 3) G. Birkhoff: Lattice Theory (1948).

2) E. Pitcher and M. F. Smiley: Transitivities of betweenness: Trans. Am. Math. Soc. 52 (1942).