ON THE LINEARIZATION OF A FORM OF HIGHER DEGREE AND ITS REPRESENTATION

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In the theory of invariants, an n-ary form of degree m $\sum a_{i_1 i_2 \dots i_m} x^{i_1} x^{i_2} \dots x^{i_m}$ $(i_1, i_2, \dots, i_m = 1, 2, \dots, n)$ is treated, only symbolically, in the linearized form

$$\sum_{i_1, i_2, \dots, i_m = 1}^{n} a_{i_1 i_2 \dots i_m} x^{i_1} x^{i_2} \dots x^{i_m} = (\sum_{i=1}^{n} a_i x^i)^m ,$$

where a_i are mere symbols, satisfying the relations

$$a_i a_k = a_k a_i \, , \ a_{i_1} a_{i_2} \dots a_{i_m} = a_{i_1 i_2 \dots i_m} \text{ [1]}.$$

And, in the theory of spinors, an n-ary quadratic form $a_{ij} x^i x^j (a_{ij} = a_{ji})$ is linearized by the quantities γ_i satisfying $\gamma_{(i} \gamma_{j)} = a_{ij}$, in the form

$$\sum_{i,j=1}^{n} a_{ij} x^i x^j = (\sum_{i=1}^{n} \gamma_i x^i)^2 ,$$

and the structure and representation of the Clifford algebra generated by these γ_i have been investigated by many authors.[2]

We wish to extend the theory of spinors, by linearizing the n-ary form of degree m $\sum a_{i_1 i_2 \dots i_m} x^{i_1} x^{i_2} \dots x^{i_m}$ by the quantities p_i in the form $\sum a_{i_1 \dots i_m} x^{i_1} \dots x^{i_m}$ $= (\sum_{i=1}^{n} p_i x^i)^m$, and by investigating the structure and representation of the algebra generated by these p_i. In this paper, we shall define the generalized Clifford algebra by extending the concept of the ordinary Clifford algebra, and consider the linearization of $\sum_{i=1}^{n} (x^i)^m$ and its representation, by means of the particular case of this algebra.

However, the above quantities p_i satisfy the relations $p_i p_k = \omega p_k p_i$, different from the Weitzenböck's symbols a_i. But, from the standpoint of the theory of invariants, this fact does not come into question.

§1. Generalized Clifford Algebra

We shall define a generalized Clifford algebra (briefly G. C. algebra), by extending the concept of the ordinary Clifford algebra.[3]

1) R. Weitzenböck, Invarianten Theorie, (1923), p. 3.

2) R. Brauer and H. Weyl, Spinors in n dimensions, Amer. J. Math., vol. 57 (1935), pp. 425-449; C. Chevalley, Theory of Lie groups, (1946), p. 61.

3) C. Chevalley, ibid.