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On Integrally Closed Noetherian Rings

By

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Introduction. An interesting characterization of integrally closed Noetherian integral domains by the notion of symbolic powers was obtained by Prof. S. Mori and T. Dodo in the following form.

THEOREM (M). Let $\mathfrak{o}$ be a Noetherian integral domain with a unit element. If $\mathfrak{o}$ is integrally closed, then it follows that

i) Every prime divisor of any principal ideal $(a)(\neq(0), \neq\mathfrak{o})$ is minimal, and

ii) Every primary ideal which belongs to any minimal prime ideal $\mathfrak{p}$, is a symbolic powor of $\mathfrak{p}$.

Conversely, if the following condition iii) is satisfied, $\mathfrak{o}$ is integrally closed.

iii) If $\mathfrak{p}$ is any prime divisor of any principal ideal then there exists no primary ideal between $\mathfrak{p}$ and $\mathfrak{p}^{(2)}$.[1)]

In this note we extend this theorem to the case where $\mathfrak{o}$ is not free from zero divisors. The main purpose of this paper is to prove the following

THEOREM 1. *Let $\mathfrak{o}$ be a Noetherian ring with a unit element, and let K be its total quotient ring. Assume first $\mathfrak{o}$ is integrally closed in K. Then*

i) *Let $\mathfrak{p}$ be any prime divisor of any regular principal ideal $(a)(\neq\mathfrak{o})$. Then $\mathfrak{p}$ contains properly only one prime ideal and this prime ideal is a primary component of the zero ideal.*

ii) *Let $\mathfrak{p}$ be any minimal regular prime ideal in $\mathfrak{o}$, then every primary ideal which belongs to $\mathfrak{p}$ is a symbolic power of $\mathfrak{p}$.*

Conversely, if $\mathfrak{o}$ satisfies the following condition iii), *$\mathfrak{o}$ is integrally closed.*

iii) *If $\mathfrak{p}$ is any prime divisor of any regular principal ideal, then there exists no primary ideal between $\mathfrak{p}$ and $\mathfrak{p}^{(2)}$.*

Our proof is entirely based on the so-called primary ideal theorem and device of forming quotient rings.

Conventions of terminology. Let $\mathfrak{o}$ be a Noetherian ring and let K be its total quotient ring. If $\mathfrak{a}$ is an ideal in $\mathfrak{o}$, we call prime divisors of $\mathfrak{a}$ the prime ideals which occur as associated prime ideals of the primary ideals in a shortest representation of $\mathfrak{a}$ as an intersection of primary ideals.

Non zero divisor of $\mathfrak{o}$ shall be called regular. We shall call an ideal $\mathfrak{a}$ in $\mathfrak{o}$

1) S. Mori und T. Dodo, *Bedingungen für ganze Abgeschlossenheit in Integritätsbereichen*, This Journal 7 (1937) 15-28.