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On Numerical Integration of Ordinary Differential Equations

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§ 0. Introduction.

For numerical integration of the ordinary differential equations, there are various formulas, but these formulas are divided into two classes. The one is the class of the formulas for integrating ahead by extrapolation, and the other is the class of the formulas for checking and improving the approximate values found by the former formulas. In this paper, we call the former the extrapolation formulas and the latter the improving formulas. Now, all the extrapolation formulas, except for Runge-Kutta's, are obtained by integrating Newton's interpolation formula over some intervals, and the improving formulas by integrating Newton's or central-difference interpolation formula over some intervals. However, the improving formulas based on central-difference formula are used only when the approximate valus are found sufficiently ahead. Thus, except for Runge-Kutta's formula, all the formulas of both classes used in the first step are based on Newton's interpolation formula.

Thus, in this paper, at first, we integrate Newton's interpolation formula over intervals of arbitrary numbers. Next we consider the general linear combination of the formulas thus obtained and seek for the accurate formulas more convenient for practical use than the customary ones. Namely we seek for the coefficients of the linear combination so that the obtained formulas may not contain the difference of higher orders and moreover not lose their accuracy.

In this paper, we consider the differential equations of the first order and those of the second order. For the equations of the higher order, the similar reasonings will prevail.

§ 1. Integration of Newton's interpolation formula.

Newton's interpolation formula is written as follows:

$$(1.1)\quad f(x)=f_0+\frac{u}{1!}\nabla f_0+\frac{u(u+1)}{2!}\nabla^2 f_0+\cdots+\frac{u(u+1)\cdots(u+p-1)}{p!}\nabla^p f_0+S_{p+1},$$

where $x=x_0+uh$, h being the breadth of an interval. For the remainder S_{p+1}, we have:

$$(1.2)\qquad S_{p+1}=\frac{u(u+1)\cdots(u+p)}{(p+1)!}h^{p+1}f^{(p+1)}(\xi),$$