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On Homogeneous Ideals of Graded Noetherian Rings

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Introduction

Given a polynomial ring over a field K, $A = K[x_0, x_1, \cdots, x_n; y_0, y_1, \cdots, y_m]$ we may indicate by $A_{i,j}$ the set of forms of degree i in the x, j in the y. Then A is the direct sum of K-submodules $A_{i,j}$:

$$A = \sum_{0 \leqq i, j < \infty} A_{i,j},$$

and we have moreover the relation: $A_{i,j} A_{k,l} = A_{i+k,\, j+l}$. Thus this ring may be regarded to have a graded structure in a sense.

Let us note here another ring which has a similar graded structure. Given a commutative ring R, and an ideal $\mathfrak{a}$ of R such that $\bigcap_{n=1}^{\infty} \mathfrak{a}^n = (0)$, we form the formring [(1)] of the ideal $\mathfrak{a}$:

$$F(\mathfrak{a}) = \sum_{n=0}^{\infty} \mathfrak{a}^n / \mathfrak{a}^{n+1},$$

where holds again the relation:

$$(\mathfrak{a}^n / \mathfrak{a}^{n+1}) \cdot (\mathfrak{a}^m / \mathfrak{a}^{m+1}) = \mathfrak{a}^{n+m} / \mathfrak{a}^{n+m+1}.$$

As is well known, this ring plays an important role in the theory of local rings.

The object of this paper is to introduce the notion of graded Noetherian rings, which include above mentioned manyfold projective coordinate rings as well as formrings, and to formulate some elementary properties of homogeneous ideals of such rings.

DEFINITION. Let A be a commutative ring with a unit element, and J be an ordered additive semi-group with zero element 0. (J is linearly ordered

(1) For the precise definition, see e. g. P. Samuel: Algèble Locale, mémorial des sc. Math., 1953.