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On Lorentz Transformations and Continuity Equation of Angular Momentum in Relativistic Quantum Mechanics.

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§1. Introduction and outlines.

In special relativity and relativistic quantum mechanics, the fundamental laws of mechanics are formulated so as they are form-invariant under the transformations of the general Lorentz group. The transformation of the Lorentz group is obtained by means of a suitable combination of spatial rotations of the axes of coordinates in two systems together with a special Lorentz transformation of the form:

$$x'=\frac{x-ut}{\sqrt{1-u^2/c^2}}, \quad y'=y, \quad z'=z, \quad t'=\frac{t-ux/c^2}{\sqrt{1-u^2/c^2}} \tag{1.1}$$

where x, y, z, t and x', y', z', t' are space-time coordinates in two systems K and K', the uniform velocity of K' relative to K being u along x-axis of K. The equations (1.1) represent the relations between the coordinates in K and K' where the relative velocity of K' to K is *parallel to the x-axis.* However, sometimes we shall need explicit expressions for the Lorentz transformations in a more general case where the relative velocity of K' to K is not parallel to the x-axis and where the Cartesian axes in K and K' have the same orientation (not arbitrary orientations relative to each other). Such a transformation is the so-called *Lorentz transformation without rotation* [1]*). The explicit expression for such Lorentz transformation without rotation is given by the following vector form: [1]

$$\begin{aligned} \boldsymbol{X}' &= \boldsymbol{X} + \boldsymbol{U}\left[\frac{(\boldsymbol{UX})}{u^2}\{(1-u^2/c^2)^{-\frac{1}{2}}-1\} - t\,(1-u^2/c^2)^{-\frac{1}{2}}\right] \\ t' &= (1-u^2/c^2)^{-\frac{1}{2}}\{t-(\boldsymbol{UX})/c^2\} \end{aligned} \tag{1.2}$$

where

$$\boldsymbol{X} = (x, y, z), \quad \boldsymbol{X}' = (x', y', z'), \quad \boldsymbol{U} = (u^1, u^2, u^3),$$

*) The ciphers in the square brackets refer to the Bibliography attached to the end of this paper.