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On the Non-Commutative Solutions of the Exponential Equation $e^x e^y = e^{x+y}$ II

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In our previous paper [2] [1], we have obtained all the non-commutative solutions of $e^x e^y = e^{x+y}$ for the complex algebras of degree two, and then, as the special cases, for the ternary algebras of degree two, the quaternion algebra and the total matric algebra of order two.

In this paper, we shall first consider the non-commutative solutions of $e^x e^y = e^{x+y}$ for certain iteration pairs, which, as a special case, contains the case for the complex algebras of degree two, secondly for the algebra composed of the triangular matrices of order n, and finally we shall obtain all the non-commutative solutions of $e^x e^y = e^y e^x = e^{x+y}$ for the total matric algebra of order three.

I. NON-COMMUTATIVE SOLUTIONS OF $e^x e^y = e^{x+y}$ FOR CERTAIN ITERATION PAIRS

1. Preliminary. Let $\mathfrak{A}$ be an algebra with the unit 1 over the complex field $\mathfrak{C}$, and let us denote by Greek letters $\alpha, \beta, \ldots, \lambda, \mu, \ldots$ the elements of $\mathfrak{C}$.

We shall consider the set $\mathfrak{S}$ of all the pairs (x, y) of elements $x, y \in \mathfrak{A}$ such that

$$x^3 = \alpha^2 x,\ y^3 = \beta^2 y,\ x^2 y = y x^2 = \alpha^2 y \text{ and } x y^2 = y^2 x = \beta^2 x, \tag{1.1}$$

where $\alpha, \beta \in \mathfrak{C}$. Here it is easily seen that $(x, x+y) \in \mathfrak{S}$ is equivalent to $(y, x+y) \in \mathfrak{S}$, and for this it is necessary and sufficient that $x(x+y)^2 = \gamma^2 x$ and $y(x+y)^2 = \gamma^2 y$. But, in the following we shall never assume that $(x, x+y) \in \mathfrak{S}$. We shall investigate the non-commutative solutions of $e^x e^y = e^{x+y}$ for the set $\mathfrak{S}$.

For $(x, y) \in \mathfrak{S}$, the following are easily seen:

$$e^x = 1 + p(\alpha) x + r(\alpha) x^2 \tag{1.2}$$

where

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1) Numbers in brackets refer to the referenences at the end of the paper.