JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 18, No. 1 JUNE, 1954

On Numerical Integration of the Differential Equation $y^{(n)}=f(x, y)$

By

Minoru URABE and Hiroki YANAGIHARA

(Received Feb. 5, 1954)

§ 1. Introduction.

The formulas usually used for numerical integration of the differential equation of the form

$$(E) \qquad y^{(n)}=f(x, y) \qquad (n\geqq 2)$$

are deduced in the following way:

first, to integrate Newton's backward interpolation formula over n intervals $[x_{-n+1}, x_0]$, $[x_{-n+2}, x_0]$, $\cdots$, $[x_{-1}, x_0]$, $[x_0, x_1]$ or $[x_{-n}, x_0]$, $[x_{-n+1}, x_0]$, $\cdots$ $[x_{-1}, x_0]$;

next, to make a linear combination of the above n integrated formulas so that the terms of the derivatives may not appear.

The formulas obtained in this way are more convenient for integration of the differential equation of the form (E) than the formulas containing the derivatives, but the former is inferior to the latter in accuracy since the expansion of the errors in each step of numerical integration is greater in the former than in the latter. [(1)]

In this paper, following the method of the paper (P), we make a general linear combination of the formulas obtained by integrating Newton's interporation formulas over several intervals, and we seek for the coefficients of the linear combination so that the obtained formula may not contain the terms of the derivatives and moreover be as accurate as possible.

We consider the equation of the second order and that of the third order. For the equations of the higher order, the similar reasonings will prevail.

§ 2. Integration of Newton's interpolation formula.

If we put

$$(2.1) \qquad U_\rho=\frac{u(u+1)\cdots\cdots(u+\rho-1)}{\rho\,!},$$

Newton's backward interpolation formula is written as follows:

---

1) The reason for this phenomenon will be seen in the paper: M. Urabe and T. Tsushima, *On Numerical Integration of Ordinary Differential Equations*, this journal, 17(1953), 193-219. In the following, we denote this paper by (P).