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On the Different Theorem in Complete Fields with Respect to a Discrete Valuation

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§ **1.** Throughout this note we shall be dealing with finite separable extensions K of a field k which is complete under a discrete valuation. As usual, we shall denote by $\mathfrak{O}$, $\mathfrak{P}$, $\mathfrak{K}$ and $\mathfrak{o}$, $\mathfrak{p}$, $\mathfrak{k}$ the rings of integers, prime ideals, and residue class fields in K and k, respectively. The trace from K to k will be denoted by S. Then the *different* $D(K/k)$ of K/k is defined as the inverse of $\mathfrak{M}$ such that $\lambda \in \mathfrak{M} \Leftrightarrow S(\lambda\mathfrak{O}) \subset \mathfrak{o}$.

In this note, we shall show a proof of the different theorem different from the usual one.[1)]

§ **2.** Throughout this note, let $f(x)$[2)] be the canonical defining polynomial of $\theta \in \mathfrak{O}$ in $\mathfrak{o}[x]$, and $f'(x)$ the derivative of $f(x)$ with respect to x. Then, as is well known, the different $f'(\theta)$ of θ is divisible by $D(K/k)$. In particular, if, for example, k, K are $\mathfrak{p}$-adic number fields, $D(K/k)$ is the greatest common divisor of $(f'(\theta))$ for all θ in $\mathfrak{O}$. But, in our case, it is not always true.

LEMMA 1. *The following three statements are equivalent:*

A) $\mathfrak{O}=\mathfrak{o}[\theta]$

B) $D(K/k)=(f'(\theta))$

C) There exists an element $\eta \in \mathfrak{O}$ whose residue class modulo $\mathfrak{P}$ is a primitive element of $\mathfrak{K}/\mathfrak{k}$ and one of the prime elements of $\mathfrak{P}$ in $\mathfrak{O}$ belongs to $\mathfrak{o}[\eta]$.

PROOF. We have already proved[3)]: A)$\Leftrightarrow$B). Here we shall show A)$\Leftrightarrow$C). We may take an element θ which satisfies the conditions required in C) and is one of the primitive elements of K/k. Then, if $[\mathfrak{K}:\mathfrak{k}]$[4)] $=f$, we can choose $1, \theta, \cdots, \theta^{f-1}$ as a

1) See: E. Artin, Algebraic Numbers and Algebraic Functions I, Princeton Univ. and New York Univ., 1950–1951, Chap. 5, The Different, Theorem 2.

2) $f(x)$ is irreducible in $\mathfrak{o}[x]$ with the leading coefficient 1.

3) See: A. Kinohara, A Note on the Relative 2-Dimensional Cohomology Group in Complete Fields with Respect to a Discrete Valuation, this Journal, **18**, p. 2 (1954), Lemma 1.

4) $[\mathfrak{K}:\mathfrak{k}]$ denotes the degree of $\mathfrak{K}/\mathfrak{k}$.