JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 19, NO. 3, JANUARY, 1956

Application of Majorized Group of Transformations to Ordinary Differential Equations with Periodic Coefficients

By

Minoru URABE

(Received July 16, 1955)

1. Introduction. In the previous paper[1], by means of majorized group of transformations, we have proved the following two theorems:

Theorem 1[2]. *Given an analytic transformation*

$$T: \; x_i' = \varphi_i(x) = \lambda_i x_i + [x]_2,$$

where $|\lambda_i| = 1$ *and* $[x]_2$ *denotes a sum of the terms of the second and higher orders with respect to* x_j. *Then there exists a set of analytic functions* $f_i(x)$ *of the form*

$$(*) \qquad f_i(x) = x_i + [x]_2$$

satisfying the relation

$$(**) \qquad f_i(\varphi) = \lambda_i f_i(x),$$

if either of the following two conditions is fulfilled:

1° *a set of* $\{T^k\}$ $(k = 0, \pm 1, \pm 2, \cdots)$ *is majorized, namely there exists a set of analytic functions* $\Phi_i(x)$ *such that* $\varphi_i(x, k) \ll \Phi_i(x)$, *where* $\varphi_i(x, k)$ *are the functions such that* $x_i' = \varphi_i(x, k)$ *represents a transformation* T^k;

2° *the arguments of* λ_i*'s are all commensurable with* 2π *and there exist formal series* $f_i(x)$ *of the form* (*) *satisfying the relation* (**) *formally.*

A set of functions $f_i(x)$ *meeting the requirement is given by*

$$f_i(x) = \lim_{n\to\infty} \frac{1}{n} \sum_{p=0}^{n-1} \frac{1}{\lambda_i^p} \varphi_i(x, p).$$

In the case 2°, *the above functions can be written in the finite form as follows:*

$$f_i(x) = \frac{1}{q} \sum_{p=0}^{q-1} \frac{1}{\lambda_i^p} \varphi_i(x, p),$$

---

1) M. Urabe, *Application of majorized group of transformations to functional equations.* J. Sci. Hiroshima Univ., Ser. A, **16**, 267–283 (1952). In the sequel, we denote this paper by [P].

2) [P], p. 271 and p. 273.