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# *On Some Properties of Non-Compact Peano Spaces*

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## Introduction

By a Peano space we mean a locally compact, locally connected, separable metric space. If a Peano space is compact, it is called a continuous curve or Peano continuum. In §1 we shall introduce the notion "degree of a Peano space", and in §2 we shall consider characterization of non-compactness for Peano spaces by half-open or open arcs, closed in the spaces. A metric space $R$ will be called convex provided it has a convex metric $\rho\ (x, y)$, that is, for each pair of points $p$, $q$ in $R$, there exists a point $r$ such that $\rho(p, r)=\rho(r, q)=\rho(p, q)/2$. R. H. Bing and E. E. Moise proved independently that each continuous curve has a convex metric ([1], [5]). We have shown that there exists a convex metric in each Peano space ([8]). However, the metric defined in [8] is complete but not bounded. We shall show that there exists a bounded convex metric in each Peano space (in §4).

We know several compactings of non-compact spaces. In §5 we shall consider a property of the socalled Freudenthal's compacting. In the last section the problem of extension of metric will be treated.

## 1. Degree of Peano spaces.

We shall recall that each locally compact Hausdorff space $R$ has a compacting by adding a new point $\xi$; we denote the associated compact space by $R^*=R+\xi$, and then define the closure operator $—^*$ in $R^*$ as follows:

$$\bar{M}^*=\bar{M}: \text{ if } M\not\ni\xi \text{ and } \bar{M} \text{ is compact,}$$

$$\bar{M}^*=\bar{M}+\xi: \text{ if } M\not\ni\xi \text{ and } \bar{M} \text{ is not compact and}$$

$$\bar{M}^*=(\overline{M-\xi})+\xi: \text{ if } M\ni\xi.$$

Hence each open set of $R^*$ is either an open set of $R$ or $R^*-F$, where $F$