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A Method of Numerical Integration of Analytic Differential Equations

By

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1. Introduction

For the truncation error committed in the use of approximate formulas, P. Davis[1] proposed a new estimate by introducing a Hilbert space of analytic functions and using essentially the Riesz representation of bounded linear functionals. In the present note, we seek for the formulas of numerical integration of analytic differential equations such that their truncation errors may be least in the error estimate of Davis, and compare them with the traditional formulas.

2. Derivation of new formulas

We consider the formula of numerical integration of the form

$$\int_{x_0}^{x_1} f(x)\,dx = h\sum_{j=-1}^{N} a_j f(x_{-j}), \tag{1}$$

where $x_{-j} \fallingdotseq x_0 - jh$ $(h>0)$. We assume that the complex extension $f(z)$ of $f(x)$ is regular and single-valued in $|z-x_0|<\rho$ and belongs to the class H^2 there. That is to say, $f(z)$ has the Taylor expansion

$$f(z) = \sum_{n=0}^{\infty} c'_n (z-x_0)^n \qquad (\,|z-x_0|<\rho\,) \tag{2}$$

and it is valid that

$$\|f\|^2 = \int_0^{2\pi} f\bar{f}\,d\theta = 2\pi\sum_{n=0}^{\infty} |c'_n\rho^n|^2 < \infty, \tag{3}$$

θ being such that $z-x_0=\rho\exp(i\theta)$. We take h so that $|x_{-j}-x_0|<\rho$ for $j=$

1) P. Davis, *Errors of numerical approximation for analytic functions*, J. Rational Mech. Anal. **2**, 303-313 (1953).