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On a Method to Compute Periodic Solutions of the General Autonomous System

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1. Introduction

Previously [4, 8][1] the writer devised a method to compute a periodic solution for the general autonomous differential system and, as an example, applying that method to van der Pol's equation, he computed the periodic solutions for the values 0 (0.2) 1.0 of the damping coefficient [6]. But, for that method, he had to prove directly the convergence of the iterative process using somewhat troublesome estimations of various quantities, because that method was completely different from Newton's method for the solution of equations.

Recently, however, the writer found that, without causing any radical change in the actual computation, the method can be altered so that it may be reduced to solution of certain equations by means of Newton's method, consequently the derivation of the method may be greatly simplified.

This note is devoted to the explanation of this modified method.

2. Moving orthonormal system along an orbit

The modified method is also based on the variation of orbits of the autonomous system and, for the study of this, the moving orthonormal system [5, 7, 8] along an orbit (not necessarily closed) is used. So the results about the moving orthonormal system which are essential for this note are stated in this section.

Given the autonomous system

$$\frac{dx}{dt}=X(x) \tag{1}$$

where $X(x)$ is an N-times ($N\geqq 1$)[2] continuously differentiable function with respect to x in a domain G of n-dimensional Euclidean space R^n, and let

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1) The numbers in the brackets refer to the references listed at the end of the note.

2) In the actual computation, N must be not-small, because, otherwise, we could not apply any integration formula to the given system.

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