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Note on a Simple Closed Curve Bounding a Pseudo-Projective Plane

Akira Tominaga

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Introduction

A 3-*manifold* is a separable metric space each of whose points has a closed neighbourhood homeomorphic to a 3-cell. Hereafter we do not assume any conditions about connectedness, compactness, orientability and boundary for 3-manifolds except for Corollary 2 in Section 1. Any 3-manifold to be considered in the following is supposed without loss of generality to have a fixed triangulation. Moreover, every thing will be considered from the semi-linear point of view. For example, mappings are semi-linear, curves are polygonal and surfaces are polyhedral, and so on.

Let m be a positive integer and let P be a disc in euclidean plane E^2 which is mapped onto itself by each rotation r_a in E^2 defined by the equations:

$$\begin{aligned} x' &= x\cos\left(\frac{a}{m}\cdot 2\pi\right) - y\sin\left(\frac{a}{m}\cdot 2\pi\right), \\ y' &= x\sin\left(\frac{a}{m}\cdot 2\pi\right) + y\cos\left(\frac{a}{m}\cdot 2\pi\right) \end{aligned} \qquad (a=1,\cdots, m).$$

Then if we identify each pair of points p, p' on [1] Bd P such that $r_a(p)=r_{a'}(p')$ for some a, a', we have from P a regular connected 2-dimensional polyhedron P_m, called a *pseudo-projective plane*, cf. [1] p. 266. In particular P_1 is a disc and P_2 is a projective plane. The above identification defines a mapping f of P onto P_m in a natural way. Then the set f (Bd P) is a simple closed curve L in P_m and is called a *boundary curve* of P_m.

In the previous paper [5], we gave a necessary and sufficient condition that a simple closed curve in a 3-manifold M bounds a disc in M. We now show in this note a property of a boundary curve L of P_m in M (Theorem 1) and give a condition that $L\subset M$ should be a boundary curve of $P_m \subset M$ (Theorem 2). We shall make use of the method of diagrams in proving these theorems. Furthermore at the end of this note, a remark for [5] will be added.

1. Pseudo-projective planes

First we indicate several definitions for diagrams. Let D be a singular

(1) Bd means boundary.