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On Relatively Semi-orthocomplemented Lattices

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A complete lattice L is called a Z-lattice if its center Z is a complete (Boolean) sublattice of L and $\bigvee(a_\alpha;\alpha\in I)\frown b=\bigvee(a_\alpha\frown b;\alpha\in I)$ holds when $a_\alpha\in Z$ for every $\alpha\in I$ or $b\in Z$ (F. Maeda [4], Definition 1.1). If a complete lattice L has a binary relation "$\perp$" which satisfies the six axioms $(1,\alpha)$—$(1,\zeta)$ introduced in my previous paper [5], then it is a Z-lattice (see Theorem 1.3 and Lemma 1.3 of [5]). In this paper, it will be proved that the axiom $(1,\varepsilon)$ is unnecessary for the proof that L is a Z-lattice. A lattice with 0, 1 (not necessarily complete) which has a binary relation "$\perp$" satisfying these five axioms except $(1,\varepsilon)$ will be called to be relatively semi-orthocomplemented. Then, the above statement means that a relatively semi-orthocomplemented complete lattice is a Z-lattice. This is the main theorem of this paper.

We shall show that relatively orthocomplemented lattices and complemented modular lattices are relatively semi-orthocomplemented lattices with some special properties. Moreover, we shall show that, in a ring A with unity, if the set $R_I(A)$ of all principal right ideals generated by idempotents of A forms a lattice by set-inclusion, then it is a relatively semi-orthocomplemented lattice; especially that if A is a Baer ring, then $R_I(A)$, equal to the set of all right-annihilators, is a relatively semi-orthocomplemented complete lattice.

Our main theorem includes the following theorems as special cases: Theorem 2 of Loomis [3], on a relatively orthocomplemented complete lattice (see [4], Remark 4.3); Theorem 5 of Kaplansky [1], on a complemented modular complete lattice; Theorem 5.3 of F. Maeda [4], on a lattice of the annihilators of a Baer ring.

1. Definitions and examples. We assume that, in a lattice L with 0, there is a binary relation "$\perp$" which satisfies the following axioms:

$(\perp 1)$ $a\perp a$ *implies* $a=0$;
$(\perp 2)$ $a\perp b$ *implies* $b\perp a$;
$(\perp 3)$ $a\perp b,\ a_1\leqq a$ *imply* $a_1\perp b$;
$(\perp 4)$ $a\perp b,\ a\smile b\perp c$ *imply* $a\perp b\smile c$.

These axioms coincide with $(1,\alpha)$, $(1,\beta)$, $(1,\gamma)$ and $(1,\delta)$ in [5, §1] respectively. It is obvious by $(\perp 1)$, $(\perp 2)$ and $(\perp 3)$ that $a\perp b$ implies $a\frown b=0$. Two elements $a, b\in L$ are called to be *semi-orthogonal* when $a\perp b$. L is called to be *semi-orthocomplemented* if it has 1 and every element $a\in L$ has a complement $a^\perp$ such that