J. Sci. Hiroshima Univ. Ser. A
Vol. 23, No. 3, March, 1960

A Generalization of the Stroboscopic Method

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(Received September 30, 1959)

1. Introduction

In this note, we consider a real system of n nonlinear differential equations of the form as follows:

$$\frac{dx_i}{dt}=\varepsilon f_i(x, t, \varepsilon) \qquad (i=1, 2\cdots, n), \tag{1.1}$$

where $f_i(x, t, \varepsilon)$ $(i=1, 2,\cdots, n)$ are such that

1° *they are continuous with respect to* (x, t, ε) *in the domain*

$$D:\ |x|=\sum_{i=1}^{n}|x_i|<L,\ -\infty<t<+\infty,\ |\varepsilon|<\delta;$$

2° *they are periodic in* t *with the period* $T(>0)$;

3° *they are once continuously differentiable with respect to* x_i $(i=1, 2,\cdots, n)$ *and* ε.

For such a system, N. Minorsky proposed an interesting method [2]—— the so-called *stroboscopic method* —— to seek for periodic solutions and to decide their stability. After his proposal, his heuristic method was proved to be mathematically legal too by some writers [3, 4]. The stroboscopic method guaranteed mathematically advocates that, for sufficiently small $|\varepsilon|$,

1° *to each simple critical point of the system —— the so-called stroboscopic image of* (1.1) ——

$$\frac{dx_i}{dt}=\varepsilon F_i(x) \qquad (i=1, 2,\cdots, n), \tag{1: 2}$$

where

$$F_i(x)=\frac{1}{T}\int_0^T f_i(x, t, 0)dt \qquad (i=1, 2,\cdots, n), \tag{1. 3}$$

there corresponds one and only one periodic solution with period T of the initial system (1.1);

2° *the stability of the corresponding periodic solution of the initial system is the same as that of the corresponding critical point.*

Recently, R. Faure [1] considered the case where some of $F_i(x)$ $(i=1, 2, \cdots, n)$ vanish identically —— in this case, evidently there does not exist any simple critical point —— and he has found to seek for a periodic solution of the initial system making use of Haag's successive approximations. But his