JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 22, NO. 3, DECEMBER 1958

A Condition Under Which Simple Closed Curves Bound Discs

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(Received September 20, 1958)

1. Introduction

A 3-manifold is a separable metric space M such that each point of M lies in an open set whose closure is a 3-cell (a set homeomorphic to the unit sphere plus its interior in euclidean 3-space E^3). We may assume without loss of generality that M has a certain triangulation by E. E. Moise's work. Moreover, every thing will be considered from the semi-linear point of view. For example, a curve means a polygonal one.

Let N be a connected subset in M, a, b points on N and C a curve from a to b such that $C-(a+b)\subset M-N$. Let us suppose that there exists a curve C' on N joining a to b and homotopic to C in M. If the image of the interior of Q under the homotopic mapping is in $M-N$, we shall say C is *ρ-homotopic* to C' in M with respect to N, where Q is the fundamental square of the homotopic mapping. If for any a, b and C there exists such a C', we denote this fact by $\pi_1(M-N, N)=1$.

In this paper the following theorem will be proved (§ 2).

THEOREM. *Let M be a 3-manifold, compact or not, with boundary which may be empty and L a simple closed curve in M. A necessary and sufficient condition that L bounds a disc in M is that there exists a neighborhood U of L such that L is homotopic to zero in U and* $\pi_1(U-L, L)=1$.

The proof of this theorem is based on Dehn's lemma, ([2], [5], [7]) i.e. if M is a 3-manifold as in the theorem and D is a Dehn disc (§ 2) in M, then bd D[1] bounds a disc.

In the last section we shall refer to knots in 3-sphere S^3 as an application of the theorem.

2. The proof of the theorem

We shall first note several definitions used in this section. Let $\tilde{D}$ be a triangulated disc, and $f:\tilde{D}\rightarrow M$ an unhomeomorphic mapping of $\tilde{D}$ into M such

(1) bd=boundary.