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On the Nonlinear Autonomous System Admitting of a Family of Periodic Solutions near its Certain Periodic Solution

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1. Introduction

Let

$$\frac{dy}{dt}=Y(y) \tag{1.1}$$

be a given nonlinear autonomous system, where $Y(y)\in C^2_y$ in a domain G of the phase $(n+1)$-space R^{n+1}. Here, of course, y and Y are the $(n+1)$-dimensional vectors. In the sequel, let us call the independent variable t the *time*.

In this paper, we consider the case where, in a domain G, the system (1.1) admits of a family $\mathfrak{F}$ of closed paths in the neighborhood of its certain closed path

$$C_0: y=\phi(t).$$

According to the nomenclature of the preceding paper [5][1], the system (1.1) is called respectively the *fully* or the *partially oscillatory* system according as the family $\mathfrak{F}$ consists of whole paths or of a portion of them lying near C_0. In either case, according to [5], we assume that, when $n\geqq 2$, the periods of the closed paths belonging to $\mathfrak{F}$ are bounded. Then, by **4.1** of [5], there exist the *universal periods* for paths belonging to $\mathfrak{F}$ such that they are continuous at C_0. Let us denote such a universal period of C_0 by ω_0.

According to [5], we make use of a moving orthonormal system along C_0. Let ξ_i's $(i=1, 2, \cdots, n)$ be the normal unit vectors of a moving orthonormal system along C_0 such that $\xi_i(t)\in C^2_t$. Then, with respect to this moving orthonormal system, any path C lying near C_0 is represented as

$$y=\phi(t)+\sum_{i=1}^{n}x^i\xi_i \tag{1.2}$$

and the time variable τ along C and the n-dimensional vector $x=(x^i)$ are determined respectively by the differential equations

$$\frac{d\tau}{dt}=\frac{\|Y\|^2+\sum_{i=1}^{n}x^iY^*\dot{\xi}_i}{Y^*Y'}\ {}^{2)}, \tag{1.3}$$

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1) Numbers in brackets refer to the references listed at the end of the paper.

2) $\|Y\|$ denotes the Euclidean norm of Y. The dots above the letters denote differentiation with respect to t. The symbol * denotes the transposed.