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Periodic Solution of Van der Pol's Equation with Damping Coefficient $\lambda=0$ (0.2) 1.0

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1 Introduction

On a periodic solution of Van der Pol's equation

$$\frac{d^2x}{dt^2}-\lambda(1-x^2)\frac{dx}{dt}+x=0 \quad (\lambda>0), \tag{1}$$

its existence and uniqueness for any value of λ is well known [1][1)] and it is also well known [1] that the unique periodic solution is orbitally stable. For $\lambda=0.1$, 1.0 and 10, Van der Pol sought for the orbits of the periodic solutions by the graphical method [2]. From the behavior of these orbits, the properties of the periodic solutions of (1) are suggested somewhat, but, of course, the further and more minute properties of the periodic solutions are not known from these results. When λ is sufficiently small [3] or large [4], the properties of the periodic solutions are known considerably in detail, but, when λ lies between these two limits, hardly any property of the periodic solutions seems to be known yet.

In this report, in order to contribute to this respect, making use of the method of the previous paper [5], the author calculated the periodic solutions of (1) for $\lambda=0$ (0.2) 1.0, correct to three decimal places. From this result, the various properties of the periodic solutions for $\lambda=0$ (0.2) 1.0,—the shapes of the orbit and the oscillation, the amplitude and the period of the oscillation, the characteristic exponent h of the orbit, etc.—are found. The remarkable results newly ascertained compared to those of Van der Pol are as follows:

With increasing λ till 1.0,

1° the amplitude varies—probably increases—very slowly;

2° the period increases normally;

3° the stability increases very rapidly, and the sum of the characteristic exponent and the damping coefficient λ varies very slowly.

2 Methods of calculation

Writing (1) in the simultaneous form

$$\begin{cases} \dfrac{dx}{dt}=y \ (=X) \\ \dfrac{dy}{dt}=-x+\lambda(1-x^2)y \ (=Y), \end{cases} \tag{2}$$

1) Numbers in the crotchets refer to the references listed at the end of the report.