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Moving Orthonormal System along a Closed Path of an Autonomous System

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1 Introduction

As was used Poincaré [1][1] and later by some writers [2], a moving orthonormal system along a closed path[2] representing a periodic solution of an autonomous system is very convenient for study of orbital stability and perturbation of a periodic solution of an autonomous system, if such a moving orthonormal system exists. However, so long as the writer knows, the existence of such a moving orthonormal system seems to have not yet been proved for a general autonomous system, consequently the utilization of such a moving orthonormal system also seems to have been insufficient.

In this note, for a continuous autonomous system, the existence of such a moving orthonormal system having the same smoothness as that of the given autonomous system is established. But the method of constructing such a moving orthonormal system in the proof is not convenient to practical construction. So the convenient method available in the most cases for practical construction is added. Then three applications of such a moving orthonormal system—applications to the variational equation, the stability problem and the perturbation problem—are shown.

2 Existence of a moving orthonormal system

Given an autonomous system

$$\frac{d\boldsymbol{x}}{dt}=\boldsymbol{X}(\boldsymbol{x})^{3)}, \tag{2.1}$$

where $\boldsymbol{X}(\boldsymbol{x})$ is continuous in a domain G of a phase n-space R^n. Assume that (2.1) has a closed path

$$C:\boldsymbol{x}=\boldsymbol{\varphi}(t)$$

lying in G, and let the positive period of $\boldsymbol{\varphi}(t)$ be ω[4].

From definition of a path, it is assumed that

1) Numbers in the crotchets refer to the references listed at the end of this note.

2) This means that the moving system is periodic with the same period as that of the closed path and one of unit vectors of the system is tangent to the closed path.

3) Letters in Gothic type denote the vectors.

4) We call ω also the period of the closed path C.