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On Some Properties of FC-groups

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1. Introduction.

In the investigation of the infinite groups, the groups in which all the classes of conjugate elements are finite have received an attention in recent years. Such groups were named "FC-group" by R. Baer and studied by R. Baer, B. H. Neumann, J. Erdös and F. Haimo. And it is known that these groups have many similar properties with abelian groups and finite groups.

In this paper, we shall investigate some properties of FC-groups. First, we require the necessary and sufficient conditions in order that a group be an FC-group. Next we prove that in FC-groups the division hull of an arbitrary subgroup, that is, the set of all elements for which there exists a positive integer m such that the m-th power of it is contained in the subgroup, is also subgroup. This is a common property of FC-groups with abelian groups and finite groups. Moreover we show that the simple FC-groups are finite groups. Finally we prove that some FC-groups decompose in the product of a finite number of its subgroups.

We use the following notations: When S and T are two subsets of a group G, $S \cap T$ is the intersection of S and T. And $P(S)$ means the set of all elements of S each of which has a finite order, that is, the periodic part of S. Next, let S be a subgroup of G. S' is the derived group of S. And the division hull of S in G is denoted by $D(S:G)$ which due to F. Haimo [3].[1] Moreover, if a and b are two elements of G, $[a, b]$ is the commutator of a and b.

2. Characterization of FC-groups.

If G is an FC-group, then its subgroups and factor groups are FC-groups. But the converse is not true. In this section we obtain the necessary and sufficient conditions in order that a group be an FC-group. And each of them is corresponding to the converse.

First, we prove a lemma.

LEMMA 1. *If a group G has a normal subgroup N such that $N \cap G'$ is*

1) The numbers in brackets refer to the references at the end of this paper.