JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 21, NO. 1, APRIL, 1957

Note on Fixed-Point Theorem

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(Received Jan. 25, 1957)

1. Introduction

The present note deals with relations between f.p.p. (the fixed point property) for some types of continua, for example, plane continua not separating the plane (§ 4), and continua whose arcwise components, which will be defined later, are finite in number (§ 6). By the way, we shall concern with multi-valued mappings on the n-simplex (§ 5).

2. Terminology and notations[5]

We shall assume that R^n is provided with a fixed Cartesian coordinate system. Then we adopt a metric, ρ, as follows: let $x=(\xi_1,\cdots,\xi_n)$, $y=(\eta_1,\cdots,\eta_n)$ be two points. Then we define $\rho(x, y)=\max\limits_{1\leq i\leq n}(|\xi_i-\eta_i|)$. $U(x, \varepsilon)$ is an ε-neighborhood of x under ρ. Let S be a subset of R^n, then $\mathfrak{J}(S)$, $\mathfrak{F}(S)$ are the interior, and the frontier of S respectively. The empty set is designated by ϕ.

The *ε-grating*, $\mathfrak{G}_\varepsilon$, is the collection of the principals, $\xi_i=\varepsilon m$, where $\varepsilon>0$ and $m=0, \pm 1, \pm 2,\cdots$. ε will be said the *mesh* of $\mathfrak{G}_\varepsilon$. The *n-cells on* $\mathfrak{G}_\varepsilon$ are the closures of the rectangular domains into which R^n is divided by $\mathfrak{G}_\varepsilon$. The *0-cells* (or *vertices*) and *1-cells on* $\mathfrak{G}_\varepsilon$ are the vertices and the edges on $\mathfrak{G}_\varepsilon$ respectively. By a *stepped arc on* $\mathfrak{G}_\varepsilon$ is meant a simple arc which is the union of 1-cells on $\mathfrak{G}_\varepsilon$. $\mathfrak{G}_{\varepsilon/k}$ (k: an integer ≥ 2) will be called a *refinement* of $\mathfrak{G}_\varepsilon$.

In this paper a *n-polytope*, P, means a point set which is the union of a finite set of n-cells on some fixed grating, $\mathfrak{G}_\varepsilon$. A cell in P means a cell on $\mathfrak{G}_\varepsilon$ which is contained in P. A vertex in $P\subset R^2$ is called to be *singular* if it is a common vertex which belongs to precisely two 2-cells in P. Thus a singular vertex is a boundary point of P.

Let p be a singular vertex of P and $\mathfrak{G}_{\varepsilon/k}$ a refinement of $\mathfrak{G}_\varepsilon$. Now let C_1, C_2 be two 2-cells on $\mathfrak{G}_{\varepsilon/k}$ contained in P and having p as their common vertex. Then we have a new polytope $\overline{P-(C_1\cup C_2)}$. This process will be called slicing of P at p by $\mathfrak{G}_{\varepsilon/k}$.