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On the Branches of Logarithmic Function of a Matrix Variable

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§ 1. Introduction

Let $\mathfrak{A}$ be the space of all the complex matrices of degree n with the usual topology, $\mathfrak{M}$ the subspace of $\mathfrak{A}$ which is composed of all the regular matrices of degree n, and then we shall consider the exponential mapping: $W \rightarrow Z = \exp W$ from $\mathfrak{A}$ onto $\mathfrak{M}$, by $\mathfrak{L}(M)$ we shall denote the complete inverse image of M of this mapping, i.e., $\mathfrak{L}(M) = \{A; \exp A = M\}$, and by $\mathfrak{A}^{\circ}$ we shall denote the maximal subspace of $\mathfrak{A}$ for which the mapping: $W \rightarrow Z = \exp W$ from $\mathfrak{A}$ onto $\mathfrak{M}$ is locally homeomorphic. And moreover we shall set $\mathfrak{A}^s = \mathfrak{A} - \mathfrak{A}^{\circ}$ (the complement of $\mathfrak{A}^{\circ}$ with respect to $\mathfrak{A}$), $\mathfrak{M}^s = \exp \mathfrak{A}^s$ and $\mathfrak{M}^{\circ} = \mathfrak{M} - \mathfrak{M}^s$.

In the previous paper ([3])[1] we have written the following facts:

(1) $\mathfrak{A}^{\circ}$ is the subspace of $\mathfrak{A}$ composed of all the matrices whose characteristic roots λ_i do not satisfy the condition: $\lambda_i - \lambda_j = 2l\pi\sqrt{-1}$ (l: non-zero integer) ([3], Theorem 3).

(2) $\mathfrak{M}^{\circ}$ is the subspace of $\mathfrak{M}$ composed of all the matrices whose minimal polynomials are of degree n, this condition is equivalent to $\dim \mathfrak{C}(M) = n$ where $\mathfrak{C}(M)$ means the set of all the matrices commutative with M.

(3) $A \in \mathfrak{C}(M) \frown \mathfrak{A}^{\circ}$, if and only if $\mathfrak{C}(A) = \mathfrak{C}(M)$ ([3], Theorem 5).

(4) $\mathfrak{A}^{\circ}$ is open and dense in $\mathfrak{A}$, and is arc-wise connected, but is not simply connected.

For a fixed matrix M of $\mathfrak{M}$, the set $\mathfrak{L}(M)$ has been already considered in the previous papers ([1], [2] and [3]). In this paper we shall consider the set $\mathfrak{L}(Z)$ for a variable matrix Z of $\mathfrak{M}$, and by making use of the correspondence between the arcs in $\mathfrak{A}$ and the arcs in $\mathfrak{M}$ under the exponential mapping we shall investigate the behavior of the points of $\mathfrak{L}(Z)$ for Z moving along arcs.

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1) Numbers in brackets refer to the references at the end of the paper.