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Jordan and Jordan Triple Isomorphisms of Rings

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A *Jordan homomorphism* or semi-homomorphism of an associative ring $\mathfrak{A}$ into an associative ring $\mathfrak{B}$ is defined as a mapping $a \to a'$ such that

(I) $(a+b)'=a'+b'$,

(II) $(ab)'+(ba)'=a'b'+b'a'$.

In the ring $\mathfrak{B}$, if $2x'=0$ implies $x'=0$, it is called $\mathfrak{B}$ *has not additive order* 2. It is well known that, on the assumption that the additive order of $\mathfrak{B}$ is not 2, the additive mapping (II) is equivalent to the following:

(II)′ $(a^2)'=(a')^2$

and implies (III):

(III) $(aba)'=a'b'a'$.

In this paper we will consider the meanning of the mapping (III) (Theorem 1) and for the prime ring prove the generalization of G. Ancochea's theorem [1][1] (Theorem 2). Also, we will show a result similar to Jacobson-Rickart's theorem [3] for the one-to-one mapping (I), (III) (Theorem 3). Our principal result (Theorem 2) is based on the identities in Lemma 1 and 2. Recently, I. N. Herstein has proved some theorems for the Jordan homomorphisms [2]. His Theorem H is similar to our Theorem 2. The difference between his result and ours is that we do not require that the additive order of the image ring is not 3.

1. We may get the Jordan ring $\mathfrak{A}_j$ from the associative ring $\mathfrak{A}$ by introducing Jordan product $\{a, b\}=ab+ba$ for any pair of elements a, b in $\mathfrak{A}$. Then we can regard the Jordan homomorphism of $\mathfrak{A}$ into $\mathfrak{B}$ as the homomorphism of $\mathfrak{A}_j$ into $\mathfrak{B}_j$. Such relation holds for the mapping (I), (III).

THEOREM 1. *Let $a \to a'$ be an additive mapping which satisfies* (III) *of a ring $\mathfrak{A}$ into a ring $\mathfrak{B}$ of additive order different from* 2, *then it is a Jordan triple homomorphism, that is $\{\{a, b\}c\}'=\{\{a', b'\}c'\}$ for any $a, b, c \in \mathfrak{A}$. And conversely.*

PROOF. For arbitrary elements $a, b, c \in \mathfrak{A}$

$$(abc+cba)'=((a+c)b(a+c)-aba-cbc)'=a'b'c'+c'b'a'.$$

Hence, $\{\{a, b\}c\}'=(abc+cba)'+(bac+cab)'=\{\{a', b'\}c'\}$. Conversely, $2\{a^2, b\}'=\{\{a, a\}b\}'=2\{(a')^2, b'\}$. Since the additive order of $\mathfrak{B}$ is not 2, we have $\{a^2, b\}'=\{(a')^2, b'\}$. Therefore, $2(aba)'=\{\{a, b\}a\}'-\{a^2, b\}'=2a'b'a'$. Thus, this theorem is proved.

1) Numbers in brackets refer to the references at the end of the paper.