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m-Connectedness and Polyhedral Inner Approximations of Plane Peano Continua

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1. Introduction.

A plane peano continuum, M, is a set which is homeomorphic with a locally connected, connected compact (nonvoid) set in the plane π. Though its topological structure is generally clear, we shall somewhat notice the topological type in the large. The principal apparatus of investigation for M is cyclic element theory. We arrange plane peano continua in accordance with the notion "m-connectedness". Furthermore, for M we prove the equivalency of "simply connected", being equal to "1-connected", and some other notions, notably the equivalency of the topological notion "shrinking" and the metrical one "existence of the unique geodesic segment for each pair of points". The latter subject was proposed by Bing[1].

Next we intend to realise M in π as a mosaic consisting of segments and 2-simplices, i.e. to show that M is homeomorphic with a figure $\overline{\sum P_i}$ where P_i is an Euclidean polyheder[2], or, in other words, M is a *polyhedral inner approximation.*

Some of the definitions adopted in this paper are due to the Whyburn's book[3] and the Newman's one[4].

Notations for metric. Let R be a metric space with an associated distance-function $\rho(x,y)$ for $x, y \in R$. If $A \subseteq R$, $\rho(A)$ is the diameter of A for ρ, i.e. $\rho(A) = \sup\ \{\rho(x,y)/x, y \in A\}$. If $p \in R$, $\rho_p(A) = \sup\ \{\rho(p,x)/x \in A\}$. Let R be a metric space, having metric ρ, such that a pair of points, x, y, has a geodesic segment joining them. We shall denote the geodesic segment by $\langle x, y\rangle_\rho$, and denote $\langle x, y\rangle_\rho - y = \langle x, y)_\rho$ etc. d is the Euclidean metric in π. If $x, y \in \pi$, we abbreviate $\langle x, y\rangle_d$ by $\langle x, y\rangle$.

Notations for mapping. Let R, S be spaces and A a subset of R. If

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(1) R. H. Bing: Partitioning continuous curves, Bull. Amer. Math. Soc. vol. **58** (1952), pp. 536–556.

(2) An Euclidean polyheder is the union of a finite set of simplices.

(3) G. T. Whyburn: Analytic topology, Amer. Math. Soc. Colloquium Publications, vol. **28** (1942), New York.

(4) M. H. A. Newman: Elements of the topology of plane sets of points, Cambridge at the University Press (1954).