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Reduction of Periodic System to Autonomous One by Means of One-Parameter Group of Transformations

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§ 1. Introduction

Between the periodic system

$$(1.1)\qquad \frac{dx_i}{dt}=X_i(x, t)=\sum_{j=1}^{n}c_{ij}(t)x_j+\sum_{\mathfrak{p}}{}''c_{i\mathfrak{p}}(t)x_1^{p_1}\cdots x_n^{p_n}\quad (i=1, 2, \cdots, n)$$

and the autonomous system

$$(1.2)\qquad \frac{dx_i}{dt}=\xi_i(x)=\sum_{j=1}^{n}c_{ij}x_j+\sum_{\mathfrak{p}}{}''c_{i\mathfrak{p}}x_1^{p_1}\cdots x_n^{p_n}\quad (i=1, 2, \cdots, n)$$

where $c_{ij}(t)$, $c_{i\mathfrak{p}}(t)$ are continuous for $-\infty<t<\infty$ and periodic with period $\omega>0$ and $\sum_{\mathfrak{p}}''$ denotes summation over $\mathfrak{p}=(p_1, p_2, \cdots, p_n)$ where $p_1, p_2, \cdots, p_n$ are non-negative integers such that $s(\mathfrak{p})=p_1+p_2+\cdots+p_n\geqq 2$, there is known till now a considerable amount of parallel properties. But, for lack of the principle connecting the two systems, such parallel properties must have been proved till now on each system respectively, even though the proof may be carried on in parallel. In this paper, we would establish such a principle.

For this purpose, we consider to transform the system (1.1) to the system (1.2) by the transformation of the form

$$(1.3)\qquad x_i=F_i(y, t)=\sum_{j=1}^{n}k_{ij}(t)y_j+\sum_{\mathfrak{p}}{}''k_{i\mathfrak{p}}(t)y_1^{p_1}\cdots y_n^{p_n}\quad (i=1, 2, \cdots, n),$$

where $k_{ij}(t)$, $k_{i\mathfrak{p}}(t)$ are continuous for $-\infty<t<\infty$ and periodic with period ω. If this is possible, then the systems (1.1) and (1.2) correspond to each other by the correspondence (1.3), namely a principle connecting both systems is established. But, as is seen later, transformation like (1.3) is not always possible. In this paper, we show that *transformation like* (1.3) *is possible when and only when the certain finite transformation can be imbedded in a one-paramter group of transformations.*

This result can be applied to various problems. The problem of reducing the periodic system to that of the simplest form is one of them. As an example of application of our result, this problem is attacked from our standpoint. As another important application of our result, the so-called