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On Tori

Akira Tominaga

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Introduction

A *3-manifold* M is a connected, separable metric space each of whose points has a closed neighbourhood homeomorphic to a 3-cell. It is well known that each 3-manifold is triangulable. M is said to be *closed* if it is compact and(1) Bd M is empty. In the following every thing will be considered from the semi-linear point of view.

A *solid torus V of genus h* (≥ 0) means a 3-cell with h solid handles (i. e. Henkelkoerper vom Geschlechts h [10] p. 219), that is a compact orientable 3-manifold, whose boundary is a torus of genus h or equally an orientable closed surface of genus h, constructed by a 3-cell C, $2h$ mutually exclusive discs D_1, D'_1;$\cdots$; D_h, D'_h on Bd C and h homeomorphisms, f_i: $D_i \to D'_i$, of identification. The fundamental group $\pi_1(V)$ of V is a free group with h free generators. m_i $=$Bd D_i and D_i are called a *meridian* and a *meridian disc* of V, respectively. Furthermore the set $\{m_1, \cdots, m_h\}$ is called a *system of meridians of V*. We note that m_i's are mutually exclusive and are homologously independent on Bd V. A *system of longitudes*, conjugate to a system of meridians $\{m_1, \cdots, m_h\}$ of V, consists of a point p on Bd V and h simple closed curves l_i on Bd V such that $l_i \cdot l_j = p$, $l_i \cdot m_j = \emptyset$ for $i \neq j$ and l_i intersects m_i at only one point.

In this paper we are concerned with several problems of situation of tori of genus one in 3-sphere S^3 (§§ 1-3). Theorem 1 gives a topological characterization of S^3 by the situation of tori of genus one and Theorem 2 shows a n. a. s. condition that a polyhedral torus of genus $h \geq 2$ bounds a solid torus in S^3. There is a question as follows: [7] (16.4) p. 330: Let T and T' be two tori of the same genus h in S^3, such that the closure of each one of the components of $S^3 - T$ and $S^3 - T'$ is a solid torus. Does there exist an isotopy of S^3 onto itself carrying T onto T'? If $h=0$ the answer is affirmative [5]. In §3 we show that it is also affirmative for $h=1$. In the last section a characterization of systems of longitudes of solid tori is shown.

1. A topological characterization of S^3

In 1924, J. W. Alexander proved that each polyhedral torus of genus one in S^3 bounds a solid torus [1] and afterward H. Schubert gave a detailed proof of the same proposition [9], §4, pp. 151–155. Conversely we show that

(1) Bd M means the boundary of M.