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On a Ring whose Principal Right Ideals Generated by Idempotents Form a Lattice

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Introduction

In his algebraic study of operator algebras [4], Kaplansky considered a ring (resp. *-ring) with unity in which the right annihilator of any *subset* is the principal right ideal generated by an idempotent (resp. a projection = self-adjoint idempotent). Then the left annihilator of any *subset* has the similar property. He called such a ring (resp. *-ring) a Baer ring (resp. Baer *-ring).

In a ring $\mathfrak{A}$, we denote by $\mathcal{R}_I(\mathfrak{A})$ the set of all principal right ideals generated by idempotents, which is partially ordered by set-inclusion. We know that if $\mathfrak{A}$ is a Baer ring then $\mathcal{R}_I(\mathfrak{A})$ (equal to the set of all right annihilators) forms a complete lattice. On the other hand, if $\mathfrak{A}$ is a regular ring of von Neumann then $\mathcal{R}_I(\mathfrak{A})$ (equal to the set of all principal right ideals) forms a complemented modular lattice. These two rings both satisfy the following conditions:

(R_r) The right annihilator of any *element* is the principal right ideal generated by an idempotent,

(R_l) The left annihilator of any *element* is the principal left ideal generated by an idempotent.

As will be stated below, to imply that $\mathcal{R}_I(\mathfrak{A})$ forms a lattice it is sufficient that $\mathfrak{A}$ satisfies these two conditions.

When $\mathfrak{A}$ is a *-ring, we consider the following condition:

(R_r^*) The right annihilator of any *element* is the principal right ideal generated by a projection.

It is obvious that the similar condition (R_l^*) for the left annihilators is equivalent to (R_r^*). These conditions (R_r^*) and (R_l^*) were treated first by Rickart [8] in the case of B^*-algebras: a B_p^*-algebra of Rickart is a B^*-algebra satisfying (R_r^*). In this paper, a *-ring is called a Rickart *-ring if it satisfies (R_r^*), and a ring is called a Rickart ring if it satisfies (R_r) and (R_l). These rings have many examples in the literatures on operator algebras and continuous geometries, i. e., Baer *-rings, *-regular rings, B_p^*-algebras and the *-rings treated by Berberian [1, § 3] are Rickart *-rings; Baer rings and regular rings are Rickart rings. This paper is devoted to the study of Rickart rings and Rickart *-rings.

In § 1, we shall prove that if $\mathfrak{A}$ is a Rickart ring then $\mathcal{R}_I(\mathfrak{A})$ forms a lat-