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A note on a result of Lanteri about the class of a polarized surface

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Abstract. Let S be a smooth complex projective surface, H be a very ample divisor on S, and $m(S,H)$ be its class. In this short note we prove that $m(S,H) \geq H^2 + 2g(S,H) + 2$ under the assumption that $m(S,H) > H^2$ and $g(S,H) \geq 2$, where $g(S,H)$ denotes the sectional genus of (S,H). Moreover we classify (S,H) with $m(S,H) = H^2 + 2g(S,H) + 2$. This result is an improvement of a result of Lanteri.

1. Introduction

Let S be a smooth complex projective surface, H be a very ample divisor on S, and $m(S,H)$ be its class, i.e. the degree of the dual variety of S (embedded via H). Then some relations between $m(S,H)$ and H^2 have been studied by many authors (for example, [4], [5], [6], [7] and [9]). Among other things, in [6, (2.5) Proposition], Lanteri proved $m(S,H) \geq H^2 + 2g(S,H) + 1$ under the assumption that $m(S,H) > H^2$ and $g(S,H) \geq 2$. Here $g(S,H)$ denotes the sectional genus of (S,H), which is defined by the following formula.

$$g(S,H) = 1 + \frac{1}{2}(K_S + H)H.$$

In his paper, Lanteri also said that it is not known whether this result is the best possible or not (see [6, p. 85]). In this short note, we improve this inequality and we show that $m(S,H) \geq H^2 + 2g(S,H) + 2$ holds under the assumption that $m(S,H) > H^2$ and $g(S,H) \geq 2$. Moreover we classify (S,H) with $m(S,H) = H^2 + 2g(S,H) + 2$.

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