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On Morrey spaces of measures: basic properties and potential estimates

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1. Introduction

This paper deals with various estimates for the Riesz potentials and the heat kernel acting on the spaces of Radon measures μ on R^n whose total variation $|\mu|$ satisfies

$$|\mu|(B(z, r)) \leqq Cr^{n(1-1/p)} \tag{1.1}$$

for some $1 \leqq p \leqq \infty$. Here $B(z, r)$ denotes the open ball with radius r centered at $z \in R^n$ and the constant C is independent of r and z. The spaces of measures of the form $\mu = f\,dx$, with $f \in L^1_{\mathrm{loc}}$ and dx the Lebesgue measure, satisfying (1.1) are called Morrey spaces and have been extensively studied in connection with the regularity theory for weak solutions of nonlinear elliptic equations (see [1, 2, 5]). However, those measures are considered mostly on bounded domains since the main interest is in the local behavior of the densities.

In this paper we consider the measures satisfying (1.1) on the whole space R^n and discuss the boundedness of the Riesz potentials and the heat kernel acting on such measures. Our results are stated in the same way as in the well-known case of L^p spaces and seem to be more or less known to mathematical publicity. For example, Peetre [6] states a more general result than ours without proof; however, it seems to be not so easy to guess a proof of his result by reading the other parts of [6]. For this reason we give in this paper the detailed proofs of our results for later use.

The author encountered the spaces of measures satisfying (1.1) during the study of viscous vortex flow in three-dimensional space [3]. Although some parts of the results in this paper are already included in [3], we present here the full version of the results for the reader's convenience. In Section 2 we define the Morrey spaces of measures and investigate their elementary structures. In particular, we shall show that the norms of the Morrey spaces possess several properties in common with the norms of the Lebesgue spaces L^p and the Lorentz-Marcinkiewicz spaces L^p_w. It should be noticed that, contrary to the case of L^p and L^p_w spaces, the interpolation property of the Morrey spaces with respect to various bounded linear operators still remains an open problem. Section 3 is the main part of this paper, in which we discuss the boundedness of