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Oscillatory criteria for differential equations with deviating argument

Dedicated to Professor O. Boruvka on the occasion of his 90th birthday

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The aim of this paper is to give a new approach for considering the question concerning the oscillatory criteria for differential equations with deviating argument.

We will deal with the differential equation

$$L_n y(t) + h(t, y(\varphi(t)), y'(\varphi(t)), \dots, y^{(n-1)}(\varphi(t))) = 0\,, \qquad n > 1 \tag{E}$$

where $h: J \times R^n \to R$, $\varphi: J \to R$, $a_i: J \to (0, \infty)$, $i = 0, 1, \dots, n$, are continuous functions, $J = [t_0, \infty)$, and

$$L_0 y(t) = a_0(t) y(t)\,, \qquad L_i y(t) = a_i(t)(L_{i-1} y(t))'\,, \qquad i = 1, 2, \dots, n\,.$$

Under a solution $y(t)$ of (E) we will understand a solution existing on some ray $[T_y, \infty)$ and such that

$$\sup\{|y(t)| : t_1 \leqq t < \infty\} > 0 \qquad \text{for any} \quad t_1 \geqq T_y\,.$$

The following basic assumptions will be used:

1. $\int^\infty a_i^{-1}(t)dt = \infty$, $i = 1, 2, \dots, n-1$;
2. $y_0 h(t, y_0, y_1, \dots, y_{n-1}) > 0$ for all $t \in J$ and any $y_i \in R$, $i = 0, 1, \dots, n-1$, $y_0 \neq 0$;
3. $y_0 h(t, y_0, y_1, \dots, y_{n-1}) < 0$ for all $t \in J$ and any $y_i \in R$, $i = 0, 1, \dots, n-1$, $y_0 \neq 0$;
4. $\lim \varphi(t) = \infty$ as $t \to \infty$.

Definition 1. A solution $y(t)$ of (E) will be called oscillatory if there exists an increasing sequence $\{t_i\}_{i=1}^\infty$ such that $\lim_{i\to\infty} t_i = \infty$ and $y(t_i) = 0$, $i = 1, 2, \dots$. A solution $y(t)$ of (E) will be called nonoscillatory if it is not oscillatory, i.e. there exists $T_y' \geqq T_y$ such that $y(t) > 0$ or $y(t) < 0$ on the interval $[T_y', \infty)$.

It follows from the assumptions 1.–4. and from the equation (E) that to