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The law of small numbers and the limit theorem for symmetric statistics with mixing conditions

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§1. Introduction

There has been considerable and theoretical interest in how well the Poisson distribution approximates the distribution of the sums of arbitrary indicator (zero-one) variables. Results of this type, either limit theorems or quantitative estimates of the distance to a Poisson distribution, have been shown under various conditions by many authors. Janson [14] gave a sufficient condition (not of mixing type) for convergence to Poisson distribution of a sequence of sums of dependent indicator (zero-one) random variables. Chen [5] gave a general method of obtaining and bounding the error in approximating the distribution of the sums of dependent Bernoulli random variables by the Poisson distribution. Dobrushin and Sukhov [9], gave necessary and sufficient conditions for convergence to a Poisson process of infinite particle systems under the action of free dynamic (see also Willms [22] and Zessin [23]). The other investigations in this direction were conducted within the rapidly developing field of symmetric statistics. Silverman and Brown [20] have obtained Poisson limit theorems for certain sequences of symmetric statistics

$$(1.1)\qquad \sum h_k(X_{i_1}, \dots, X_{i_k}),$$

based on a sample of identically distributed independent random variables $X_1, \dots, X_n$, where h_k is a symmetric zero-one function and the summation is extended over all sets $\{i_1, \dots, i_k\}$ of distinct integers drawn from $\{1, \dots, n\}$. Barbour and Eagleson [2], [3] gave a general Poisson approximation theorem for symmetric statistics (1.1) from a sample of independent but not necessarily identically distributed random variables and with a symmetric zero-one function of k variables.

The Poisson limit theorems in the more general setting of symmetric statistics have been obtained by Mustafid and Kubo [18]. They have obtained the asymptotic distribution of the sums of symmetric statistics

$$(1.2)\qquad \sum_{1 \le s_1 < \cdots < s_k \le n} h_k(X_{n,s_1}, \dots, X_{n,s_k}),$$