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# Positive generalized white noise functionals

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## §1. Introduction

Let $\mu$ be the measure of Gaussian white noise (with variance 1) defined on the space $E^*$ of all real tempered distributions. That is, the characteristic function $C(\xi)$ of $\mu$ is given by

$$C(\xi) \equiv \int_{E^*} e^{i\langle x, \xi\rangle} d\mu(x) = \exp\left[-\frac{1}{2}\int_{R} |\xi(u)|^2 du\right]$$

where $\xi$ is an element of the real Schwartz space $E = \{$all real valued, $C^\infty$- and rapidly decreasing functions on $\boldsymbol{R}\}$ ([3] and [21]). The elements of $(L^2) \equiv \{\varphi; \int_{E^*} |\varphi(x)|^2 d\mu(x) < \infty\}$ are called *Brownian functionals* ([7]). In [6] we can see the idea of generalized Brownian functionals constructed on the theories of Sobolev spaces and Gel'fand triplets. Many authors have greatly developed the analysis of not only Brownian functionals but also generalized Brownian ones, e.g., [6] $\sim$ [8], [13] $\sim$ [17], [19], [20], [22] and [25]. Some of them treated the problem of positive generalized Brownian functionals and pointed out that it would be important in the light of quantum field theory or relating to the Feynman path integral.

In this paper we also consider positive generalized functionals in the white noise calculus, which is developed in a somewhat more general situation. That is, we prepare a real separable Hilbert space $E_0$ and a self-adjoint operator $D$ such that $D \geq 1$ and that $D^{-h_0}$ is of Hilbert-Schmidt type for some $h_0 \geq 1$, construct a Gel'fand triplet $E \hookrightarrow E_0 \hookrightarrow E^*$ by equipping $E_0$ with increasing and compatible norms $\{\|D^p \cdot\|_{E_0}\}_{p=0}^{\infty}$, and observe a triplet $(\mathscr{S}) \hookrightarrow L^2(E^*, \mu) \hookrightarrow (\mathscr{S}')$ with the characteristic functional of $\mu$:

$$C(\xi) \equiv \int_{E^*} e^{i\langle x, \xi\rangle} d\mu(x) = \exp\left[-\frac{1}{2}\|\xi\|_{E_0}^2\right], \qquad \xi \in E.$$

We show that every element of the space $(\mathscr{S})$ of test functionals has a unique continuous version on $E^*$ and the product of every two elements of $(\mathscr{S})$ again belongs to $(\mathscr{S})$ (cf. [15] and [18]). Using these results we prove that any *positive* continuous linear functional on $(\mathscr{S})$ is the linear form given by the