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Boundary limits of locally *n*-precise functions

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1. Introduction

In this note we investigate the existence of boundary limits of locally n-precise functions u on a domain G in R^n which satisfy a condition of the form:

$$\int_G \Psi(|\mathrm{grad}\, u(x)|)\omega(x)dx < \infty \tag{1}$$

with a nonnegative measurable function ω on G and a positive nondecreasing function Ψ on the interval $(0, \infty)$; for the definition and basic properties of locally p-precise functions, see Ohtsuka [4] and Ziemer [5]. The function $\Psi(r)$ is assumed to be of the form $r^n\psi(r)$, where $\psi(r)$ is a positive nondecreasing function on the interval $(0, \infty)$ satisfying the following conditions:

(ψ_1) There exists $A > 0$ such that

$$A^{-1}\psi(r) \leqq \psi(r^2) \leqq A\psi(r) \qquad \text{for any} \quad r > 0\,.$$

(ψ_2) $\int_0^1 \psi(r^{-1})^{-1/(n-1)}r^{-1}dr < \infty$.

For example,

$$\psi(r) = [\log(2 + r)]^\alpha,\ [\log(2 + r)]^{n-1}[\log(2 + (\log(2 + r)))]^\alpha,\ \dots,$$

satisfy the above conditions, as long as $\alpha > n - 1$.

We shall first show that if $\int_G \Psi(|\mathrm{grad}\, u(x)|)dx < \infty$, then there exists a continuous function u^* on G such that $u^* = u$ a.e. on G, and furthermore, in case G is a Lipschitz domain, u^* can be extended to a continuous function on $G \cup \partial G$.

Next, in section 3, we are concerned with the existence of limits at a given boundary point ξ, in the case where u satisfies (1) with $\omega(x) = \lambda(|x - \xi|)$ for a positive nondecreasing function λ on the interval $(0, \infty)$. Then, in the next section, we study the existence of boundary limits along certain subsets of G for a function u satisfying (1) with $\omega(x) = \lambda(\rho(x))$, where λ is as above and $\rho(x)$ denotes the distance of x from the boundary ∂G.

In the last section, we discuss the existence of limits at infinity, in case G is unbounded and $\omega \equiv 1$.