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An elementary proof of the rationality of the moduli space for rank 2 vector bundles on P^2

Takashi MAEDA

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0. Introduction. Let k be an algebraically closed field of characteristic zero and let $M(0, n)$ be the moduli space of stable vector bundles of rank 2 with chern classes $c_1 = 0$ and $c_2 = n$ on the projective plane P_k^2 over k. W. Barth showed that the function field of $M(0, n)$ over k is rational (= purely transcendental) of dimension $2n$ over a certain field F which is rational of dimension $2n - 3$ over k and hence $M(0, n)$ is a rational variety of dimension $4n - 3$ over k for all $n \geq 2$ [1]. However, M. Maruyama pointed out later that there was a gap in his proof of the rationality of the field F [4]. For an odd integer n, the rationality of $M(0, n)$ is proved by G. Ellingsrud and S. Strømme by a different method [2]. As for an even integer n, I. Naruki showed that when $n = 4$, the field F is rational over k and hence $M(0, 4)$ is a rational variety over k [5]. For even integer $n \geq 6$, many people have pointed out that the rationality of F is reduced to that of the moduli space $M_{g,hy}$ of hyperelliptic curves of genus $g = (n - 2)/2$ [3] by the descent theory of vector bundles.

However, in this paper we shall give an elementary proof of the rationality of the field F for all integers $n \geq 3$.

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1. Now we shall explain the above field F. Let $K = k(x_1, \dots, x_n, y_1, \dots, y_n)$ be a field of $2n$ variables $x_1, \dots, x_n, y_1, \dots, y_n$ and let W_n be the group of semi-direct product of S_n and $H_n = \bigoplus^n (Z/2Z)$:

$$1 \to H_n \to W_n \to S_n \to 1 ,$$

where S_n is the symmetric group of degree n which acts on H_n as permutations of direct factos.

Let $G = SL(2, k) \times W_n$ act on K as follows:

$$x_i^g = \alpha x_i + \beta y_i , \qquad y_i^g = \gamma x_i + \delta y_i \qquad \text{for} \quad g = \begin{pmatrix} \alpha\beta \\ \gamma\delta \end{pmatrix} \in SL(2, k) ,$$

$$x_i^\varepsilon = \varepsilon_i x_i , \qquad y_i^\varepsilon = \varepsilon_i y_i \qquad \text{for} \quad \varepsilon = (\varepsilon_1, \dots, \varepsilon_n) \in H_n \quad (\varepsilon_i = \pm 1) ,$$

$$x_i^\sigma = x_{\sigma(i)} , \qquad y_i^\sigma = y_{\sigma(i)} \qquad \text{for} \quad \sigma \in S_n .$$