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Positive solutions of singular Emden-Fowler type systems

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1. Introduction

In this paper we will discuss positive solutions of the singular Emden-Fowler type system

$$(1) \qquad \begin{cases} y'' = a(t)z^{-\lambda}, \\ z'' = b(t)y^{-\mu}, \end{cases} \qquad t \geq t_0,$$

where λ, $\mu > 0$ are constants, and a and b are continuous functions on $[t_0, \infty)$. The following conditions are always assumed to hold:

(C_1) a and b have unbounded supports.

(C_2) The improper integrals

$$A(t) \equiv \int_t^\infty a(s)ds \quad \text{and} \quad B(t) \equiv \int_t^\infty b(s)ds$$

converge for $t \geq t_0$, and $A(t)$, $B(t) \geq 0$, $t \geq t_0$.

(C_3) AB has unbounded support.

A vector function $(y, z) \in C^2[t_0, \infty) \times C^2[t_0, \infty)$ is called a positive solution of system (1) when it solves system (1) and $y(t)$, $z(t) > 0$ for $t \geq t_0$.

The singular Emden-Fower type equations of the form

$$y'' = h(t)y^{-\lambda}, \qquad t \geq t_0,$$

with $\lambda > 0$, $h \in C[t_0, \infty)$, have been treated in several papers; see [1-5]. Especially, the author [5] showed under suitable conditions that this equation admits a positive solution $y(t)$ satisfying

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} y'(t) = 0.$$

Sufficient conditions for the uniqueness of such solutions were also given. However, it seems that very litttle is known about such decaying solutions of system (1). Therefore in this paper we will give sufficient conditions which ensure the existence of positive solutions (y, z) of (1) satisfying