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Planar Navier-Stokes flows in a bounded domain with measures as initial vorticities

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Introduction

Let D be a simply connected bounded domain in $\mathbb{R}^2$ with smooth boundary S. In this paper we consider the two-dimensional Navier-Stokes equations of the following form :

$$\text{(NS)}\qquad \begin{aligned} &\frac{\partial u}{\partial t} + u\cdot\nabla u = \lambda\Delta u - \nabla p &&(x \in D,\ t > 0\\ &\nabla\cdot u = 0 &&(x \in D,\ t \geqq 0)\\ &u\cdot\nu|_S = 0\ ;\ \nabla\times u|_S = 0\ ;\ u|_{t=0} = a, \end{aligned}$$

and discuss the existence and uniqueness of strong solutions when the initial vorticity $\nabla \times a$ is very singular. Here, $\lambda > 0$ is the kinematic viscosity ; ν is the unit outward normal to the boundary ; $u = (u^1, u^2)$ and p are, respectively, unknown velocity and pressure ; a is a given initial velocity ; and $\nabla\cdot u = \Sigma_j \partial_j u^j$, $u\cdot\nabla u = \Sigma_j u_j \partial_j u$, $\nabla \times u = \partial_1 u^2 - \partial_2 u^1$, $\partial_j = \partial/\partial x_j$. Our goal is to establish the existence of a smooth global solution in the case where $\nabla \times a$ is a finite Borel measure on D. Our result extends those of [4, 10] obtained for the Cauchy problem to the case of simply connected bounded domains. The boundary condition for u described above not only appears in a free-boundary problem for the Navier-Stokes equations, but also is well known as a standard boundary condition for the magnetic field in the theory of magnetohydrodynamics [18].

As a byproduct we obtain an existence result for the Euler equations

$$\text{(E)}\qquad \begin{aligned} &\frac{\partial u}{\partial t} + u\cdot\nabla u + \nabla p = 0 &&(x \in D,\ t > 0)\\ &\nabla\cdot u = 0 &&(x \in D,\ t \geqq 0)\\ &u\cdot\nu|_S = 0\ ;\ u|_{t=0} = a, \end{aligned}$$

in the case where $\nabla \times a$ belongs to L^q for some $q > 1$, by investigating the behavior of solutions u_λ to (NS) as λ goes to 0. A similar result was obtained by Bardos [2] in L^2-framework, and our result can be regarded as an L^p-version