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Stationary solutions of a reaction-diffusion equation with a nonlocal convection

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Abstract: We are concerned with an ecological model described by a nonlinear diffusion equation with a nonlocal convection. The conditions under which stationary solutions exist are investigated. We also discuss the stability problem of stationary solutions.

1. Introduction

Reaction-diffusion equations are widely used in the modelling in biology, chemistry and other fields. Kawasaki [3] has proposed an ecological model described by a nonlinear diffusion equation with a nonlocal convection. The model of this type has been further studied by Nagai & Mimura [5], Mimura & Ohara [4], Ikeda [2] in the whole line of $\boldsymbol{R}^1$, whereas Ei [1] has considered the model in the finite interval. In the latter case the equation of interest takes the form

$$u_t = u_{xx} - [(K * u)u]_x + F(u), \qquad x \in I = (-1/2, 1/2) \tag{1.1}$$

subject to the boundary condition

$$u_x - (K * u)u = 0 \qquad \text{at } x = \pm 1/2 \tag{1.2}$$

and the initial condition

$$u(x, 0) = u_0(x) \geq 0, \qquad x \in I. \tag{1.3}$$

Here $u = u(t, x)$ denotes the population density at time t and the position x. The convection term $[(K * u)u]_x$ corresponds to aggregating mechanism of the population, where $(K * u)(x) = \int_I K(x-y)u(y)\,dy$ and $K(x)$ is an appropriate odd function satisfying $K(x) < 0$ for $x > 0$.

A representative kernel $K(x)$ is

$$K(x) = \begin{cases} \gamma e^{\beta x} & (x < 0), \\ -\gamma e^{-\beta x} & (x > 0), \end{cases} \tag{1.4}$$

where γ, β are nonnegative constants. One knows that when