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Asymptotic behavior of solutions to certain nonlinear parabolic evolution equations II

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1. Introduction

The purpose of this paper is to show that given a pair of solutions u_1, u_2 in $W^{1,1}_{\text{loc}}((0, \infty); H)$ of the time-dependent evolution equation

$$(1.1) \qquad (d/dt)u(t) + \partial\psi^t(u(t)) \ni 0, \qquad a.e.\ t \geq 0,$$

the strong convergence

$$(1.2) \qquad \mathrm{s} - \lim_{t\to\infty}\{u_1(t) - u_2(t)\} = \text{const.} \in H$$

is valid, where H is a real Hilbert space, and for each $t \in [0, \infty)$, ψ^t is a proper lower semi-continuous (l.s.c.) convex functional defined in H and $\partial\psi^t$ denotes the subdifferential of ψ^t.

A typical example of (1.1) is the following parabolic equation:

$$(1.3) \qquad \begin{cases} (\partial/\partial t)u(t, x) - \sum_{j=1}^{n}(\partial/\partial x_j)f_j(t, x, \nabla u) + g(t, x, u) = 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad (t, x) \in \bigcup_{t\geq 0}\{t\} \times Q(t), \\ u(t, x) = 0, \qquad\qquad\qquad\qquad\quad (t, x) \in \bigcup_{t\geq 0}\{t\} \times \Gamma(t). \end{cases}$$

Here, for each fixed (t, x), the family $\{f_j(t, x, y)\}$ is supposed to be completely integrable with respect to $y \in \mathbf{R}^n$ and an ellipticity condition

$$(1.4) \qquad \sum_{j,k=1}^{n}(\partial/\partial y_j)f_k(t, x, y)\ \xi_j\xi_k \geq r(t)a(x)|\xi|^2, \qquad \xi \in \mathbf{R}^n$$

holds for some positive smooth functions r on $[0, \infty)$ and a on $\mathbf{R}^n$. For each $t \in [0, \infty)$, the set $Q(t)$ denotes a domain in $\mathbf{R}^n$ with smooth compact boundary $\Gamma(t)$. In most of our results, we do not assume the boundedness of $Q(t)$. By means of the zero-extension, we formulate equation (1.3) in the real Hilbert space $L^2(\mathbf{R}^n)$.

The convergence (1.2) is interesting, for example, if $\partial\psi^{t+T} = \partial\psi^t$ with some $T > 0$ and $u_2(.)$ is a T-periodic solution of (1.2). The existence of periodic solutions of our example (1.3) is obtained in [5] (see also [7]).

As mentioned in the Introduction of our previous paper [4], to get the strong convergence (1.2) we need the following lemma.