HIROSHIMA MATH. J.
23 (1993), 629–644

Global existence of bifurcating solutions to a two-box prey-predator model

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(Received May 8, 1992)

§ 1. Introduction

P. Waltman [1] considered the following prey-predator system:

$$(1.1)\qquad \begin{cases} \dfrac{ds(t)}{dt} = \gamma s(t)\left(1 - \dfrac{s(t)}{K}\right) - \dfrac{1}{k}\dfrac{ms(t)}{s(t) + a_1}x(t) \equiv f(s, x) \\ \dfrac{dx(t)}{dt} = \left(\dfrac{ms(t)}{s(t) + a_1} - D\right)x(t) \equiv g(s, x) \\ s(0) = s_0 > 0\,, \qquad x(0) = x_0 > 0\,, \end{cases}$$

where $x(t)$ denotes the population of the predator, $s(t)$ the population of the prey, m the maximum growth rate of the predator, D the death rate of the predater, a_1 the half-saturation constant of the predator, k the yield factor of the predator feeding on the prey, γ the intrinsic rate of increase for the prey and K the carrying capacity for the prey population. The parameters m, a_1, k, D, γ and K are all positive constants.

In this model, the prey grows logistically in the absence of predation and the predator consume prey according to a saturating functional response. It is well known that the solutions of (1.1) are positive and bounded and that the system (1.1) has three typical behaviors: (a) Dominance. When the value of carrying capacity K of prey is less than λ ($\lambda = a_1 D/(m - D)$), the critical point $(K, 0)$ of (1.1) is asymptotically stable, namely in this case as $t \to \infty$ the prey grows to its limited value and predator becomes extinct. (b) Coexistence. When $\lambda < K < a_1 + 2\lambda$ the critical point (λ, x^*) ($x^* = k\gamma(a_1 + \lambda)(K - \lambda)/(mK)$) of (1.1) is asymptotically stable. (c) Periodicity. When $K > a_1 + 2\lambda$ system (1.1) has a stable periodic orbit in the first quadrant of the s-x plane (see [3], [4]).

On the other hand, the basic interest of the present paper is in the study of spatial and spatio-temporal patterns of the population densities when spatial migration effect of the two species are introduced in the model. If linear diffusions in space are assumed to represent spatial migration effects, the model would be a system of reaction-diffusion equations. The behaviors of