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Kirillov-Kostant theory and Feynman path integrals on coadjoint orbits of a certain real semisimple Lie group

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0. Introduction

Alekseev, Faddeev and Shatashvili showed in [1] that any irreducible unitary representation of compact groups can be obtained by path integrals. They computed characters of the representations. We showed in [3] that path integrals give unitary operators of the representation which is constructed by Kirillov-Kostant theory for the Heisenberg group, the affine transformation group on the real line, $SL(2, \mathbf{R})$ ($\cong SU(1, 1)$) and $SU(2)$. For the affine transformation group, we took a real polarization, for $SU(2)$ a complex polarization (but computed without Hamiltonians), and for the Heisenberg group and $SL(2, \mathbf{R})$ both a real polarization and a complex polarization. (For a complex polarization of $SL(2, \mathbf{R})$, we realized it as $SU(1, 1)$ and computed without Hamiltonians.)

In [4] we found that, in order to compute the path integrals with nontrivial Hamiltonians for $SU(1, 1)$ and $SU(2)$ to obtain unitary operators realized by Borel-Weil theory, we have to regularize the Hamiltonian functions, and in [5] we extended the results to the case that the maximal compact subgroup K of a connected semisimple Lie group G has equal rank to the complex rank of G.

In this paper we work with a linear connected noncompact semisimple Lie group G and consider real polarizations.

Let $\mathfrak{g}$ be the Lie algebra of G. We fix a Cartan involution θ of $\mathfrak{g}$ and let the corresponding Cartan decomposition [6] be

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p} .$$

Let $\mathfrak{a}$ be a maximal abelian subalgebra of $\mathfrak{p}$ and $\mathfrak{m}$ the centralizer of $\mathfrak{a}$ in $\mathfrak{k}$. If we fix a notion of positivity for $\mathfrak{a}$-roots, we can let $\mathfrak{n}$ be the nilpotent subalgebra given as the sum of the root spaces for the positive roots.

In this paper, we explicitly compute the path integrals with Hamiltonians for $Y \in \mathfrak{m} \oplus \mathfrak{a}$ or $\mathfrak{n}$, to give unitary operators of the representation which is constructed by Kirillov-Kostant theory. When we compute the path integral with the Hamiltonian for $Y \in \mathfrak{n}$, we make the following assumption.