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Structure of the probability contents inner boundary of some family of three-parameter distributions

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1. Introduction

Let $F(x)$ be a strictly increasing and continuously differentiable distribution function (d.f.) on the real line $\boldsymbol{R}$, and let $h(x)$(resp. $\tilde{h}(x)$) be a continuous and strictly increasing function on $\boldsymbol{R}_+ = (0, \infty)$(resp. $\boldsymbol{R}$) with $h(\boldsymbol{R}_+) = \boldsymbol{R}$(resp. $\tilde{h}(\boldsymbol{R}) = \boldsymbol{R}$). Define a transformation $t(x, \theta)(\theta = (\alpha, \beta, \lambda) \in \boldsymbol{R}_+ \times \boldsymbol{R} \times [-\infty, \infty))$ by

$$t(x, \theta) = \begin{cases} \alpha\tilde{h}(x) - \beta, & \lambda = -\infty, \\ \alpha h(x - \lambda) - \beta, & \lambda \neq -\infty. \end{cases}$$

Let Θ be a nonempty subset of $\boldsymbol{R}_+ \times \boldsymbol{R} \times [-\infty, \infty)$ and put $\mathscr{F}(\Theta) = \{F(t(x, \theta)); \theta \in \Theta\}$, being called a family of three-parameter d.f.'s which are positive only to the right of a shifted origin. The family $\mathscr{F}(\boldsymbol{R}_+ \times \boldsymbol{R} \times \boldsymbol{R})$ with $h(x) = \log x$ was considered in Finney [4].

Suppose that:

(i) We have N different kinds of experiments on some characteristic X.

(ii) The transformed variable $t(X, \theta)$ has a d.f. F.

(iii) In the ith experiment, n_i objects are tested and information available for each characteristic $X_{ij} (1 \leq j \leq n_i)$ is only that its value lies in a proper subinterval $\mathscr{C}_{ij}$ of $\boldsymbol{R}$ with nonempty interior.

The collection $\mathscr{C} \equiv \{\mathscr{C}_{ij}; 1 \leq i \leq N, 1 \leq j \leq n_i\}$ is called a *pooled interval-censored* (p.i.c.) *data.* When $N = 1$, the p.i.c. data $\mathscr{C}$ is simply called an *interval-censored* (i.c.) *data.* The i.c. data $\mathscr{C}$ is called a *grouped data* if each $\mathscr{C}_{1j}$ belongs to a set of mutually disjoint intervals whose union is equal to $\boldsymbol{R}$. The p.i.c. data $\mathscr{C}$ is called a *binary response data* if each $\mathscr{C}_{ij}$ belongs to a set of mutually disjoint two intervals, depending only on i, whose union is equal to $\boldsymbol{R}$.

There are various kinds of method for estimating the unknown true parameter θ_0 based on the p.i.c. data $\mathscr{C}$ (cf. [2], [9], [13]). In these methods, an estimate $\hat{\theta}$ of the unknown true parameter θ_0 is defined by an optimal solution of a minimizing problem. Hence there arises a problem whether such